Superior Knowledge, Price Discrimination, and Customer Inspection

Abstract

Firms in many industries may obtain superior knowledge of customer preferences, whereas customers often need costly efforts to learn their match values. In this paper, we examine the optimal pricing strategies for a firm with superior knowledge, when customers can reduce information asymmetry by costly inspection. On the surface, it seems that the firm can directly communicate a customer's match value through personalized prices, thus there is no need for customers to expend inspection efforts. Contrary to this intuition, we find that personalized pricing cannot fully obviate customer inspection. In equilibrium, the firm may trick low-preference customers into overpaying, even when inspection costs are low. This opportunistic incentive arouses customer suspicion, which then induces customer inspection that would be avoided if the firm were not capable of personalized pricing. Since inspection cost raises a deadweight loss in social welfare, public policies that prevent firms from price discriminating against customers may benefit both firms and customers.

Key words: Superior Knowledge, Price Discrimination, Customer Inspection, Customer Privacy, Signaling.

1. Introduction

Customers are often uncertain about their personal match value of novel products or credence services. For instance, gluten-free food is commonly considered as healthy diets that reduce weight, yet studies show that such diets may also cause nutritional deficiencies or even weight gain in a considerable number of people (Jones 2017). Moreover, customers often lack the knowledge to fully assess their need for credence goods (e.g., financial and legal services and auto-part repairs and replacements; for example, novice drivers are often unsure about whether their vehicle requires an engine crankcase flush). Additionally, in business-to-business markets, downstream firms may be unsure about how to evaluate technology products due to either organizational obstacles or market insensitivities. For example, Pang (2018) finds that U.S. Air Force procurement departments commonly misevaluate medical equipment.

To accurately assess their match values, customers may inspect products before making a purchase decision. For example, health-conscious customers may seek consultation at clinics to determine whether they can benefit from gluten-free diets; drivers may consult an expert or read online reviews before deciding to purchase auto repairs; and procurement departments may invest more on market research to achieve better trades with suppliers. These efforts cost either time, labor, or money. Earlier studies have investigated these costs theoretically, empirically, and experimentally (Wathieu and Bertini 2007; Guo and Zhang 2012; Li et al. 2019; Cao and Zhang 2021). We refer to the process whereby customers retrieve their match values through costly efforts as *customer inspection*. This term is intended to be understood broadly and to capture both external (e.g., information searching) and internal efforts (e.g., deliberation) that customers expend to learn about their intrinsic preferences.

By contrast, firms may obtain accurate knowledge to identify customer preferences and price discriminate, thanks to their information advantages from industry experiences and data analytics. For example, health food providers such as Unimeal and Healthline ask targeted customers for information on their dining behavior, physical fitness, and health exam records, and analyze this data to customize meal plans and offer tailored price discounts. Similarly, providers of credence goods can use technological know-how to better assess customer match values. In business-to-business markets, a supplier that serves many clients may develop a deeper understanding of its clients' needs for the relevant products. In these situations, a firm may obtain superior knowledge to identify customers about their intrinsic match value before customers learn it from inspection.

In addition, the development of data analytics technologies and growth of data vendor market have further increased the information disparity between firms and customers. For instance, online dating platforms may use the demographic data collected from their customers and behavior data purchased from data vendors (Heilweil 2020) to assess customer values. By comparing a customer's information with the pooled data of similar customers, dating sites can then better assess a customer's success rate of getting a match and offer them premium dating services at a tailored price.[1]

To the best of our knowledge, no prior research has examined the strategic interactions between customer inspection and superior knowledge, even though both are ubiquitous. To fill this gap, we study the personalized pricing strategy with superior knowledge when customers may engage in inspection. This analysis is not trivial. On one hand, when a firm has superior knowledge of its customers' preferences, it may signal this information via personalized pricing, which should reduce the customers' incentives for inspection efforts. On the other hand, firms' superior knowledge may induce customer suspicion of being subjected to higher degrees of exploitation (Furman and Simcoe 2015; Xu and Dukes 2019; 2021). If a customer believes that the firm is capable to identify her value with superior knowledge and steer her into overpaying, she should have more incentives to reduce the information asymmetry by engaging in inspection before purchase. Therefore, the impact of superior knowledge on customer inspection is subject to a formal investigation.

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[1] While most dating sites charge a uniform price for standard dating services (e.g., browsing the profiles of candidates), many also charge personalized prices for premium private dating services at a tailored price. See https://www.jiemian.com/article/6365738.html (in Chinese) for an example.

It is also unclear whether price discrimination can sustain an equilibrium when customer inspection is possible. With uniform pricing, a firm may offer either a transgressive price to induce customer inspection or a regressive price to induce no-brainer purchases (Wathieu and Bertini 2007). With personalized pricing, however, a firm prefers offering the transgressive price only to the identified higher-value customers. Therefore, anyone who receives a regressive price may update their belief and thus refrain from making a no-brainer purchase. Furthermore, conventional wisdom holds that a firm's superior knowledge facilitates price discrimination and thus helps expand the market. However, it is *a priori* unclear if this effect is beneficial when customers are suspicious of being overly exploited by superior knowledge. If customers choose to inspect more frequently in response to personalized pricing, then social welfare suffers a deadweight loss.

This paper builds a game-theoretic model to examine how firms with superior knowledge price their products or services under customer inspection. We consider a market in which a monopolistic firm sells a product or service to customers whose match value for that product is either high or low: this value is observed by the firm (e.g., through data analytics) but is not immediately known by customers. Nevertheless, customers can pay an inspection cost to find out their true preference for the firm's product. The firm tailors its price offer to each customer according to her preference, and the customer, upon observing the price, makes her inspection and purchasing decisions. Using this setting, we examine the firm's optimal pricing strategies and the customers' strategic inspection decisions and evaluate how the magnitude of the inspection cost may affect the firm, the customers, and social welfare. We also examine how a firm should manage customer inspections and improve profits and whether public policies that regulate price discrimination and protect customer welfare may holistically benefit all parties.

Specifically, we address the following research questions that are relevant to firms' use of superior information for price discrimination. First, how should a firm exploit its superior knowledge to tailor its price offers to customers who can find out their preferences through costly inspections? For example, can the firm fully exploit customer surplus through first-degree price discrimination? Second, upon observing a personalized price, will a customer trust the firm and make a no-brainer purchase of its product, or is she more likely to inspect the product first and make a more informed purchasing decision later? Third, how does the cost of inspection affect the firm and its customers? Does price discrimination always benefit the firm at the customers' expense? Should policymakers regulate price discrimination and/or data collection to protect customers and social welfare and, if so, how would such regulation affect firms? Fourth, how can a firm manage customer inspections and improve profits through decisions other than pricing?

Our findings shed light on the implications of customer inspection on firms with superior knowledge. First, while the firm can always implement first-degree price discrimination when the inspection cost is nil, it can never do so under a positive inspection cost. This result arises because, when the firm honestly charges customers their match value for a product, all customers will make a no-brainer purchase, giving the firm an incentive to deviate and charge low-preference customers a high price to trick them into overpaying for its product. In equilibrium, when inspection costs are not overly high, the firm always charges high-preference customers a high price while randomizing its price offer to low-preference customers — i.e., it sometimes charges her an honest low price and, at other times, offers her a high price to trick her into overpaying for the product. Rational customers should account for the firm's opportunistic incentives when making their inspection and purchasing decisions. Specifically, upon observing a low price, a customer knows for certain that she is a low-preference customer and makes a no-brainer purchase; meanwhile, upon observing a high price, she becomes uncertain of her customer type and randomizes between inspecting and making an informed purchase decision and making a no-brainer purchase. Only when inspection costs are high, the firm offers all customers a uniform price, and all customers make a no-brainer purchase. To our best knowledge, the novel role of the randomized price discrimination in customer inspection is not seen in earlier literature.

Second, we show that the inspection cost has non-monotone effects on a firm's profit; that is, the firm's profit is maximized when inspection is either costless or excessively expensive. When the inspection cost is zero, the firm can implement first-degree price discrimination to extract maximum value from its customers. When the cost is too high to afford for them, inspection is not a viable option for customers, and the firm can extract customer value by charging all customers a uniform price that equals their expected valuation. The firm's profit is minimized when the inspection cost is moderate, at which point price distortions become too severe. By contrast, under personalized pricing, customer surplus is always zero (i.e., regardless of the inspection cost). Consequently, total social welfare is also non-monotone with the inspection cost.

Third, in contrast to Xu and Dukes (2021) that suggest that superior knowledge benefits a price-discriminating firm, we show that, when customer inspection is possible, a firm does not necessarily benefit from its superior knowledge, even if obtaining this knowledge has no direct cost to the firm. This implies that, regulations banning data collection and/or price discrimination may actually benefit firms, which contradicts with the claims that such restriction from the existing legislation serves only the consumers (Oxera 2017). The intuition for this result is as follows. Superior knowledge gives a firm an opportunistic incentive to trick low-preference customers into overpaying for its product. In anticipation of the firm's opportunistic behavior, rational customers have suspicion of overpaying a high price and therefore choose to inspect. Therefore, the firm must distort its prices down to compensate for customer inspection. Under certain circumstances, this price distortion becomes so severe that the firm is worse off with superior knowledge and price discrimination. By contrast, if public policies ban price discrimination, customers need not worry about the firm's information abuse, which alleviate the firm's burden of price distortion. Therefore, regulations restricting price discrimination may ultimately benefit the firms. As average customer surpluses are also improved by reducing costly efforts of inspection (which are deadweight losses), such regulations may lead to a "win-win" outcome in which both the firm and customers are strictly better off.

Lastly, we discuss alternative tactics that a firm may employ to alleviate customer suspicion and thus improve profits. In an extension of our basic model that only considers pricing decisions, we find that the firm may benefit from downward distorting its product quality and offering an inferior product, even if increasing quality is costless. This is because an inferior product has higher quality-adjusted inspection cost, which may further deter customers from inspection.

2. Related Literature

This paper contributes to the growing literature on pricing strategies when firms acquire information about their customers' willingness to pay for a product or service. For example, firms frequently use information technologies to collect data on customers' purchase histories, identify past and new customers, and condition their price offers accordingly — this practice is known as *behavior-based pricing* (Fudenberg and Villas-Boas 2006). Acquisti and Varian (2005) consider a monopolistic firm selling a repeat-purchase product to customers over two periods. They find that, under behavior-based pricing, the firm will offer a higher price to repeat customers and a lower price to new customers in the second period. As a result, forward-looking customers are reluctant to buy in the first period, forcing the firm to reduce its first-period price to encourage customers into making an initial purchase. This ratchet effect hurts the firm, thus leaving it worse off when employing behavior-based pricing. Investigating a competitive market, Fudenberg and Tirole (2000) use a two-period model in which two firms, in the second period, distinguish between past customers and customers who bought from each's competitor and charge these two groups different prices. More specifically, firms charge repeat customers a higher price than what is offered to new customers. They show that, as each firm tries to poach its rival's customers, behavior-based pricing leads to an intense price competition in the second period, which leaves both firms worse off than if they had used uniform pricing. By contrast, other studies have found that behavior-based pricing may improve firm profits under certain circumstances, such as when the market comprises of both loyal customers and switchers (Chen and Zhang 2009), when customers' preferences change over time and their demands are heterogeneous (Shin and Sudhir 2010), when customers are heterogeneous in their costs to serve (Shin et al. 2012), when customers are fairness-minded

(Li and Jain 2016), when customers are ex-ante uncertain about a product's value (Jing 2016), or when firms are vertically differentiated (Rhee and Thomadsen 2017).

Firms may also go beyond behavior-based pricing to better recognize customers' preferences and offer customized prices according to their willingness to pay, a practice known as *personalized pricing*. Thisse and Vives (1988) first consider a scenario in which two competing firms can offer customers personalized prices according to their locations and show that personalized pricing intensifies the price competition, eroding both firms' profits; nonetheless, each firm cannot help but to implement personalized pricing, leading to a form of the prisoner's dilemma. Chen et al. (2001) consider a setting in which competing firms practice imperfect targeting technologies to recognize their customers. They show that, with imperfect targeting, a firm improving its targeting technologies not only benefits itself but also its rival. Chen and Iyer (2002) show that, in a competitive market, symmetric firms may choose asymmetric levels of customer addressability to soften market competition. Chen et al. (2017) study a case in which firms target customers according to their real-time geo-location, and customers can travel across different locations to get better offers. They find that, compared with traditional pricing, mobile geo-targeting can improve a firm's profit. Hajihashemi et al. (2020) analyze the impact of network effects on price personalization, and find that in markets with network effects, personalized pricing may decrease demand and profit.

The studies discussed thus far assume that customers privately know their own preferences and that firms can reduce or eliminate their information disadvantage through data collection. Research also investigates scenarios where firms have an information advantage over customers regarding their preferences. For example, for credence goods such as financial and legal services, medical care, and auto-part replacements, sellers are experts who know customers' needs, whereas customers usually lack the necessary expertise to fully assess their needs. Wolinsky (1993) demonstrates that in credence goods markets, cheating can be eliminated when customers search for multiple opinions regarding the product or when experts have reputation concerns. Fong (2005) provides a theory of cheating by an expert selling credence goods: he shows that cheating arises when customers suffer from the same problem to different extents, or when some customers' problems are costlier to solve. Jiang et al. (2014) extend Fong's theory by considering two-dimensional information asymmetry in which an expert privately observes both the customer's type and her own type (ethical or self-interested). They show that the provider's pricing decision depends on the customer's inference about both dimensions of asymmetric information. Moreover, Jing (2011) shows that when customers observe the quality of a product, within a certain range, a mixed-strategy equilibrium exists in which an informed firm selectively cheats low-type customers by recommending an unnecessary high-type product to them, and customers mix in their purchasing decisions when they are offered a high-type product. Xu and Dukes (2019, 2021) consider a case in which firms have superior knowledge of customer preferences through information aggregation. Their model assumes a common environmental noise — which they call the "market state" — under which customers receive noisy signals regarding their preferences. Consequently, only some customers are informed of their match values. These two studies discuss firms' pricing and price line design problems. While our research also assumes that the firm has an information advantage over customers regarding their preferences, we extend the literature by assuming that customers can discover their true match values through costly inspection. Thus, upon observing a personalized price from a firm, a customer decides not only whether to purchase, but also whether to inspect the product to find out her true match value.

Furthermore, our research contributes to the literature on customer deliberation — which refers to the costly cognitive process that customers must undergo to find out their valuations for certain products. Most studies consider deliberation to be independent from rational inferences such as signaling. Stigler (1966) illustrates that a high price is more likely to induce customer inspection relative to a low price. Shugan (1980) discusses the importance of modeling deliberation when studying marketing decisions. Wathieu and Bertini (2007) formally analyze a firm's pricing strategies in the presence of a deliberation cost and, supporting their findings with experimental evidence, argue that the firm may adopt either transgressive pricing to induce customer deliberation or regressive pricing to prevent it. Li et al. (2019), on the other hand, study firms' channel strategies when customers must incur a deliberation cost to find their preferences and show that customer deliberation may reduce or even eliminate the issue of double marginalization. Cao and Zhang (2021) develop a model of demand forecasting based on customer deliberation and support their theory with a large-scale field experiment. Guo and Wu (2012) and Guo (2016) study customer deliberation in the context of signaling, in which customers need to either infer the product quality or rationalize their behavioral biases before spending costs on deliberation. The key difference between our research and the foregoing studies is that, in the latter, price has the dual function of affecting a customer's purchase decision and swaying her deliberation decisions. In our model, the price has three roles: In addition to the dual roles, it signals a customer's match value.

The most related work to our model of customer inspection is Guo and Zhang (2012). They consider a firm that sells products to customers who need costly deliberation to learn about their marginal willingness to pay for quality. The firm has no additional information about customer preferences beyond the prior distribution. They show that the firm may price-discriminate against customers by offering them a menu of vertically-differentiated products. Our model departs from Guo and Zhang (2012) by assuming a different information structure: Firm has superior knowledge beyond the prior distribution to identify each customer and thus can offer personalized price to each customer, which may serve as a signal of customer preference. Therefore, the mechanism of Guo and Zhang (2012) is a form of *second-degree price discrimination*, whereas our model extends *first-degree price discrimination*. The distinction is not trivial when the firm has superior knowledge. Specifically, product menu is a public information observable by all customers, whereas personalized prices in our model is only partially observed by the targeted customer. Because of the model differences, our results also differ from those of Guo and Zhang (2012). In Guo and Zhang (2012), when the deliberation cost is low, the firm offers a product menu to customers, and all customers deliberate and make an informed purchase decision. In equilibrium, no customers overpay for their chosen product. In our model, however, when the inspection cost is low, the firm may offer a low-preference customer a high price, and upon receiving a high price, a customer makes a no-brainer purchase with a positive probability. Thus, in equilibrium, some low-preference customers overpay for the product, i.e., they pay a price that is higher than their valuation. Finally, we show that the firm can be worse off with price discrimination, which extends the classic framework (Wathieu and Bertini 2007; Guo and Zhang 2012) of uniform pricing under consumer deliberation.

Overall, we extend the literature by examining the interactions among superior knowledge, personalized pricing, and customer inspection.

3. The Model

In this section, we introduce a stylized model of price discrimination with superior knowledge against customers who can engage in inspection. The model consists of a monopolistic firm selling a product or service to a continuum of customers of unit measure. In the model, customers have heterogeneous match values v_i with the firm's product, which are either high (h) or low (l). The prior probability of a high match value is α. We refer to customers with high and low match values as high-preference and low-preference customers, respectively, and use $i \in \{H, L\}$ to denote customer i's type.[2] The firm's unit production cost is normalized to 0.[3] Both the firm and customers are risk-neutral and maximize their expected payoff.

Information Structure

We assume that, while customers know their expected match value $\bar{v} = \alpha h + (1-\alpha)l$, they are *ex ante* uninformed about their idiosyncratic shock and hence do not

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[2] Our model also admits alternative interpretations. For example, while we assume that there is a continuum of customers in the markets, our model also applies to markets with one or a few customers (e.g., credence goods and business-to-business markets): in the model, a lawyer only serves a few potential clients with heterogeneous needs for consultation services. In this case, each customer's value for the firm's product or service is high with probability α and low with probability $1-\alpha$.

[3] Our basic model implicitly assumes that the two types of customers are equally costly to serve. In the Online Appendix C, we generalize the model to consider the case in which these types of customers have difference service costs, and show that our main results will continue to hold.

know their type or their preference for the firm's product beyond its prior distribution. To resolve her preference uncertainty, a customer must exert investigation efforts such as searching for product information, recalling past experiences, simulating potential product usage scenarios and consulting an expert. Customer inspection is considered independent of rational inferences. We capture the time, effort, and resources customers spent on inspection with an inspection cost $c \geq 0$.

In contrast to the customers, the firm knows both its product characteristics and the states and/or preferences of individual customers (e.g., through its expertise or by using data analytics) and can determine the customers' expected match value $\bar{v}$ and their idiosyncratic shocks, and hence knows their match value with the product. We assume that the marginal cost for the firm to profile individual customers is negligible. The firm can then offer each customer a personalized price p_i according to her match value v_i through tools such as targeted coupons.[4]

Timeline

The model unfolds in four stages. In Stage 1, nature draws each customer's match value $v_i \in \{h, l\}$, which is perfectly observed by the firm but not by the customers. In Stage 2, the firm offers each customer a personalized price p_i according to her preference (i.e., match value). The firm may randomize its price offers; in this case, it chooses the probability distribution of its price offer, and we use $\gamma(p_i, v_i) = \Pr[p_i|v_i]$ to denote the price distribution. In Stage 3, upon observing the personalized price from the firm, customer i decides whether to inspect to find out her true preference by incurring an inspection cost. If she does not incur this expense, she remains uninformed about her preference. The inspection decision can be random, in which case the customer chooses the probability of an inspection effort upon observing the price. In this case, we use $\lambda(p_i)$ to denote a customer's inspection probability upon receiving a price p_i. In Stage 4, based on the inspection outcome, if she chooses to inspect, the customer

[4] In our stylized model, the firm perfectly learns customer preferences from customer data. However, in practice, algorithms are not always perfect and may misclassify customer types, i.e., they can misclassify a high-preference (low-preference) customer as a low-preference (high-preference) customer. Our results remain qualitatively robust when the firm's profiling technology is imperfect. See Section 7.1 for details.

decides whether to make a purchase. We assume that when a customer is indifferent about whether to make a purchase, she always purchases.

4. A No Superior Knowledge Benchmark

Before solving the model, consider a benchmark scenario in which the firm does not have superior knowledge about customer preferences (e.g., the firm does not collect customer data) and, thus, cannot price-discriminate against them. In this case, the firm offers all customers a uniform price p, which cannot convey any information about customer preferences. This benchmark also captures the scenario in which price discrimination is banned such that the firm is forced to offer a uniform price to all customers, even when it has superior knowledge of customer preference. Since the case with uniform pricing has been examined in earlier literature (Wathieu and Bertini 2007, Li et al. 2019), we only briefly illustrate the reasoning as follows.

First, the equilibrium price must satisfy $l \leq p \leq h$. Now, consider the customers' decision problem: Upon observing the price p, the customer who does not inspect maintains her expected valuation $\bar{v} = \alpha h + (1-\alpha)l$ for the firm's product and makes a purchase if and only if $\bar{v} \geq p$. If the customer makes a no-brainer purchase (i.e., buy the product without inspection), her expected surplus is

$$CS_N = \bar{v} - p.$$

If the customer chooses to inspect, she makes a purchase if and only if her match value with the firm's product is high. In this case, the customer's expected surplus is

$$CS_I = -c + \alpha(h - p).$$

The customer then compares CS_I against CS_N and the outside option, which we normalize to zero without loss of generality, and decides whether or not to inspect. That is, when $CS_I \geq CS_N$ and $CS_I \geq 0$, the customer inspects and makes a purchase when her preference is high; when $CS_N \geq CS_I$ and $CS_N \geq 0$, the customer makes a no-brainer purchase. Finally, when $CS_I < 0, CS_N < 0$, the customer neither inspects nor purchases (i.e., no-brainer no-purchase). Summarizing the customer's decision, we generate the following lemma.

***Lemma 1**. Consider the no superior knowledge benchmark. If* $p \leq \min\left(l + \frac{c}{1-\alpha}, \alpha h + (1-\alpha) l\right)$*, all customers make no-brainer purchases. If* $l + \frac{c}{1-\alpha} < p \leq h - \frac{c}{\alpha}$*, all customers inspect and make a purchase when their preferences are high. If* $\max\left(\alpha h + (1-\alpha) l, h - \frac{c}{\alpha}\right) < p$*, all customers make no-brainer no-purchases.*

Lemma 1 suggests that customers' purchasing decisions hinge on the price and inspection cost. When the price is low enough, customers make no-brainer purchases because the benefit of inspection is outweighed by its cost. Similarly, when the price is high enough, customers reject purchase without inspection. Finally, when the price is moderate, customers inspect to make a more informed purchasing decision. Note that this condition exists only if $c < \alpha(1-\alpha)(h-l)$. Otherwise, when the inspection cost is excessive, customers never inspect regardless of the price offered by the firm.

Consider now the firm's pricing decision. It either charges a price higher than the *a priori* expected value, which we call a *transgressive price*, to induce high-preference customers' inspection, or charges customers a price lower than the *a priori* expected value, which we call a *regressive price*, to induce all customers to make no-brainer purchases.

By Lemma 1, the firm solves the following optimization problem with a transgressive price:

$$\max \ \pi = \alpha p$$

$$\text{subject to } l + \frac{c}{1-\alpha} < p \leq h - \frac{c}{\alpha}.$$

By contrast, the firm solves the following optimization problem with a regressive price:

$$\max \ \pi = p$$

$$\text{subject to } p \leq \min\left(l + \frac{c}{1-\alpha}, \alpha h + (1-\alpha) l\right).$$

With a static comparison between the above pricing strategies, we obtain the following proposition.

Proposition 1*. The equilibrium prices, firm profits and customer surplus under no superior knowledge benchmark are presented in Table 1:*

Table 1: Equilibrium Strategies under the No Superior Knowledge Benchmark

	$c < \underline{c}$	$\underline{c} \le c < \overline{c}$	$\overline{c} \le c$
Price	$h - \frac{c}{\alpha}$	$l + \frac{c}{1-\alpha}$	$\alpha h + (1-\alpha) l$
Firm profit	$\alpha h - c$	$l + \frac{c}{1-\alpha}$	$\alpha h + (1-\alpha) l$
Customer surplus	0	$\alpha(h-l) - \frac{c}{1-\alpha}$	0

Note: $\underline{c} = \frac{(1-\alpha)(\alpha h - l)}{2-\alpha}, \overline{c} = \alpha(1-\alpha)(h-l)$.

Figure 1 illustrates the equilibrium price under this benchmark. As can be seen, the equilibrium price is nonmonotone with the inspection cost c. When the inspection cost is low ($c < \underline{c}$), the firm offers a transgressive price to customers and induces customer inspections. Within this regime, the equilibrium price decreases with c because the firm has to undercut its price to compensate for customers' increased efforts. When the inspection cost is moderate ($\underline{c} \le c \le \overline{c}$), the inspection cost is too high for the firm to induce customer inspection. As such, the firm charges all customers a regressive price to prevent them from inspection. As the inspection cost increases, the firm has less incentives to distort its price for no-brainer purchases. As a result, the equilibrium price increases accordingly. Finally, when inspection cost is high enough ($\overline{c} \le c$), inspection is no longer a viable option to customers, who then never inspect regardless of the price that the firm offers. In this case, the firm charges customers their expected willingness to pay and maximizes its profit. The equilibrium price is constant with the inspection cost.

In equilibrium, customers make a positive gain only when the inspection cost is moderate ($\underline{c} \le c < \overline{c}$). Within this regime, the firm sets a regressive price to persuade customers into making no-brainer purchases. As such, customers uniformly enjoy the low price and positive surplus.[5]

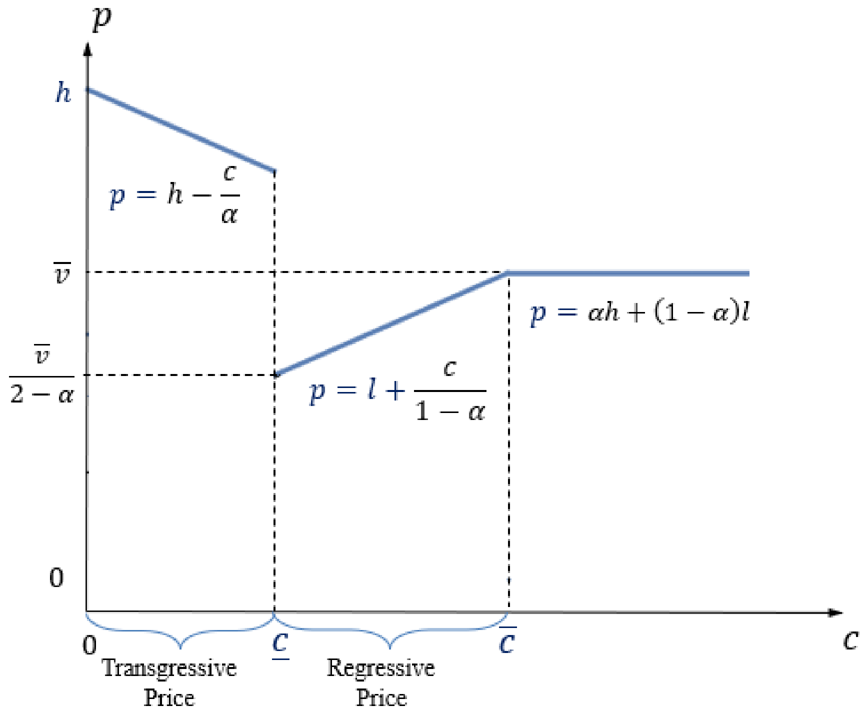

Figure 1. Equilibrium Prices in the No Superior Knowledge Benchmark

5. Model Analysis

In this section, we examine the scenario in which the firm has superior knowledge of customer preference and can therefore tailor its price offers. Because customers must incur an inspection cost to retrieve their valuation of a product, the firm enjoys an information advantage over customers and may exploit this advantage through its pricing. Thus, customers should interpret prices from the informed firm differently from how they do in the previously explored benchmark.

Because a customer does not know her own preference ex ante, the model falls into games of incomplete information, and we resort to a perfect Bayesian equilibrium (PBE) as our solution concept. In a PBE, a customer forms rational expectation about her preference for a product or service upon observing its price, which must be

[5] We refer readers to Wathieu and Bertini (2007) for a detailed discussion of equilibrium strategies that exist when the firm does not have additional information about customer preferences.

consistent with the equilibrium type along the equilibrium path. Given the customer's belief, the firm acts optimally. Because it does not place any restrictions on beliefs off the equilibrium path, a PBE often suffers from a plethora of equilibria. To pin down the equilibrium, we use the divinity criterion D1[6] (Banks and Sobel 1987; Cho and Kreps 1987).

In the analysis, we use $\Omega(p) = \Pr[v_i = h|p]$ to represent customers' posterior belief of their customer type being h when receiving price p from the firm. We use the divinity criterion D1 as our equilibrium refinement criterion. Roughly speaking, divinity criterion D1 requires a customer to disbelieve that a deviation is made by the firm to some customer types that the firm gains less often (relative to its equilibrium payoff) than to some other customer types; here, "gains less often" is evaluated by looking at the set of off-equilibrium beliefs under which the firm gains from a certain type of customer. By applying D1, we can pin down a unique equilibrium.[7]

5.1. Separating Equilibrium

We first explore the possibility for the firm to adopt a separating equilibrium. Consider first the simplest case in which $c = 0$: Here, both the firm and customers know the customers' match value, and the firm can easily implement first-degree price discrimination, offering high-preference customers the price $p_H = h$ and low-preference customers the price $p_L = l$.

But can the firm implement first-degree price discrimination under a positive inspection cost? The answer is no. To understand this, let us assume the contrary: Say there exists an equilibrium in which the firm charges high-preference customers $p_H = h$ and low-preference customers $p_L = l$. Upon observing the personalized price, the customer should learn her type with certainty. That is, p_H implies that she is identified as a high-preference customer, and p_L convinces her of the low-preference type. In

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[6] Using the intuitive criterion does not pin down the unique equilibrium in this instance.

[7] In the analysis, we assume that $\alpha \geq 1/2$. When $\alpha < 1/2$, some quantitative differences arise in the equilibrium results, but the main insights remain qualitatively the same. We discuss the case $\alpha < 1/2$ in the Online Appendix.

either case, the customer updates her belief upon observing the price and makes a no-brainer purchase. However, if such an unencumbered situation exists, the firm can deviate and also give low-preference customers a higher price p_H. Low-preference customers would then automatically believe themselves to be high-preference and make no-brainer purchases at the higher price. Now, in equilibrium, rational customers will not bend so easily: They will take the firm's opportunistic incentive into account and not purchase at the assumed prices. Thus, even if the firm has superior knowledge about customer preferences, it can never implement first-degree price discrimination. An equilibrium that allows so does not exist. We summarize this discussion in the following lemma.

Lemma 2. *For any positive inspection cost $c > 0$, there does not exist an equilibrium in which the firm implements first-degree price discrimination.*

Does, then, a separating equilibrium in which the firm offers distinct prices to different customer types exist? Proposition 2 further shows that no pure separating equilibrium exists.

Proposition 2. *For any positive inspection cost $c > 0$, there does not exist a separating equilibrium in which the firm offers different prices to different customer types.*

While the detailed proof of Proposition 2 is provided in the appendix, its intuition is as follows. Given the nature of a separating equilibrium, a customer can infer her type perfectly from the price she receives, so she need not to expend efforts to inspect. In other words, all customers will make the same purchasing decision upon observing price p: A no-brainer purchase or a no-brainer no-purchase. In either case the firm's profit is the same across all customers, which does not sustain the single crossing property of a separating equilibrium.

These analyses show that, even if the firm obtains superior knowledge of customer preferences, it cannot fully take advantage of this information to price-discriminate against customers, because the firm always has an opportunistic incentive to abuse its information and trick low-preference customers into overpaying for its product. This behavior raises customers' suspicion and makes them reluctant to make no-brainer purchases.

5.2. Pooling Equilibrium

In Section 5.1 we establish the nonexistence of a separating equilibrium whenever $c > 0$. Now, we explore the existence of a pooling equilibrium in which the firm offers the same price p to all customers. In such an equilibrium, the price does not convey any information about customer type and, therefore, we have $\Omega(p) = \alpha$, i.e., customers maintain their prior belief upon receiving the equilibrium price. We relegate the analysis to the appendix and present the equilibrium result in the following lemma.

Lemma 3. *There exists a pooling equilibrium in which the firm charges price* p *to all customers, and all customers make no-brainer purchases at this price, where*

$$p = \begin{cases} l + \frac{c}{1-\alpha} & \text{if } c \leq \bar{c}, \\ \alpha h + (1-\alpha)l & \text{otherwise.} \end{cases}$$

Note that the equilibrium strategy described in Lemma 3 is identical to the regressive pricing that was described under the no superior knowledge benchmark: The firm offers all customers a low price such that they all, regardless of type, are willing to make no-brainer purchases. There is, however, no equilibrium in which the firm universally offers customers a transgressive price. This is because a low-preference customer, upon inspection, will find out her true type and reject the high price. The firm then has an incentive to deviate, offering this customer a lower price to induce her to purchase.

While Lemma 3 establishes the existence of a pooling equilibrium over the entire parameter space, it does not always survive the divinity criterion D1 (Banks and Sobel 1987; Cho and Kreps 1987). The following proposition suggests that the pooling equilibrium exists only when the inspection cost is sufficiently high.

Proposition 3. *When $c < \bar{c} = \alpha(1-\alpha)(h-l)$, no pooling equilibrium survives the divinity criterion D1. When $c \geq \bar{c}$, there exists an equilibrium that survives the divinity criterion D1 in which the firm charges $p = \bar{v} = \alpha h + (1-\alpha)l$ to all customers.*

The proposition's detailed analysis is relegated to the appendix; however, the key intuition is as follows. When inspection cost is low, the firm can offer high-preference customers $p' = \frac{h+l+\sqrt{(h-l)(h-l-4c)}}{2} - \epsilon$, which is higher than the equilibrium pooling price. Given this price, we can show that this deviation is more likely to be profitable for the firm when offered to high-preference customers, and less likely to be profitable when offered to low-preference customers, where likelihood is evaluated at the set of out-of-equilibrium beliefs that make the deviation profitable. According to D1, customers must hold the belief that $\Omega(p') = 1$, i.e., p' is only offered to high-preference customers, who are willing to make no-brainer purchases at p'. But this gives the firm an incentive to deviate: therefore, the pooling equilibrium fails the D1 criterion when the inspection cost is not high enough.

5.3. Semi-Separating Equilibrium

We showed that there does not exist a pure-strategy equilibrium when the inspection cost is low. The intuition is that firm always has an opportunistic incentive to abuse its information advantage and trick low-preference customers into overpaying for the product, which then raise customer suspicion and affect their inspection strategy. Nevertheless, the firm may partially price-discriminate against customers using a semi-separating equilibrium. Thus, in this section, we analyze the existence of a semi-separating equilibrium in which the firm sometimes honestly charges low-preference customers according to their match value but, at others, exploits them with a high price. We defer the detailed analysis to the appendix and present our results in the following proposition. This equilibrium survives the divinity criterion D1.

Proposition 4. *When $c \leq \alpha(1-\alpha)(h-l)$, there exists an equilibrium with two prices p_1 and p_0, where*

$$p_1 = \frac{h+l+\sqrt{(h-l)(h-l-4c)}}{2},\ p_0 = l.$$

In equilibrium, the firm offers all high-preference customers price p_1 but randomizes its price offer to low-preference customers: With probability γ, it offers low-preference customers the price p_1 and, with probability $1-\gamma$, offers them the price p_0, where

$$\gamma = \frac{\alpha\left(h-l-2c-\sqrt{(h-l)(h-l-4c)}\right)}{2(1-\alpha)c}.$$

Customers' posterior belief is

$$\Omega(p) = \begin{cases} \frac{1}{2}\left(1+\sqrt{1-\frac{4c}{h-l}}\right) & \text{if } p \geq p_1, \\ 0 & \text{otherwise.} \end{cases}$$

Upon observing price p_1, the customer randomizes her inspection decision. With probability λ, she inspects and makes a purchase if and only if her match value is high. With probability $1-\lambda$, she makes a no-brainer purchase, where

$$\lambda = 1 - \frac{2l}{h+l+\sqrt{(h-l)(h-l-4c)}}.$$

Upon observing the price p_0, the customer always makes a no-brainer purchase.

Figure 2 illustrates this mixed strategy equilibrium. In the equilibrium presented by Proposition 4, a high-preference customer is always charged a high price p_1. But the firm randomly charges a low-preference customer a low price p_0 with probability $1-\gamma$ and tricks her into overpaying $p_1 > p_0$ with probability γ.[8]

Note that, if the firm tricks a low-preference customer with a high price, the customer may inspect and then reject the price after finding out her true customer type; if the firm honestly offers the customer a low price, the customer will always make a purchase. Therefore, the firm must trade-off between a high purchase likelihood and a high profit margin. In equilibrium, the firm is indifferent about which strategy to choose and, thus, willing to mix.

[8] In equilibrium, the firm may offer the same price to all low-preference customers. That is, with probability γ it offers all low-preference customers the price p_1, and with probability $1-\gamma$, it offers all low-preference customers the price p_0.

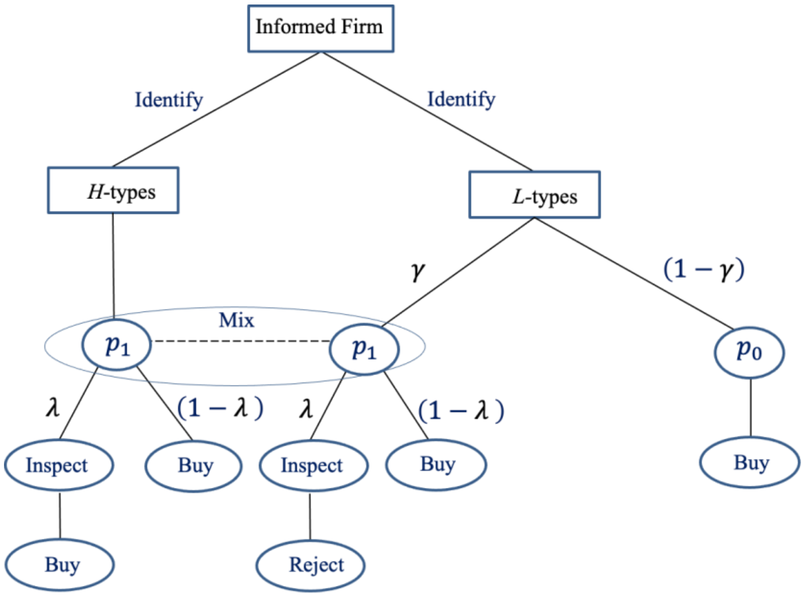

Figure 2. Game Tree of the Semi-Separating Equilibrium

Next, consider the customers' strategic choice. Upon observing a low price p_0, the customer knows with certainty that her match value is low and, given that $p_0 = l$, she is willing to make a no-brainer purchase. Upon observing a high price p_1, however, the customer becomes uncertain about her preference and may either inspect to make an informed purchase or save on the inspection cost to make a no-brainer purchase. Given the firm's pricing strategy, the customer is also indifferent about which of her two strategies to select and willing to mix.[9]

Note $p_1 < h$ in equilibrium; that is, the firm distorts the high price down and, thus, cannot fully extract its high-preference customers' value. The intuition is as follows: Given that the firm cannot help but to opportunistically trick low-preference customers into overpaying for its product, the customer becomes suspicious about her preference upon receiving p_1, i.e., $\Omega(p_1) < 1$. Then, the customer inspects with a positive probability, forcing the firm to distort the price down to compensate for the customers' inspection cost.

[9] The result that customers randomize their inspection decision hinges on the stylized assumption that all customers have the same inspection cost, and that inspection is a binary decision. Alternatively, customers may reduce inspection efforts by choosing imperfect inspection rather than random inspection if the cost function is continuous. Please see Online Appendix A for details.

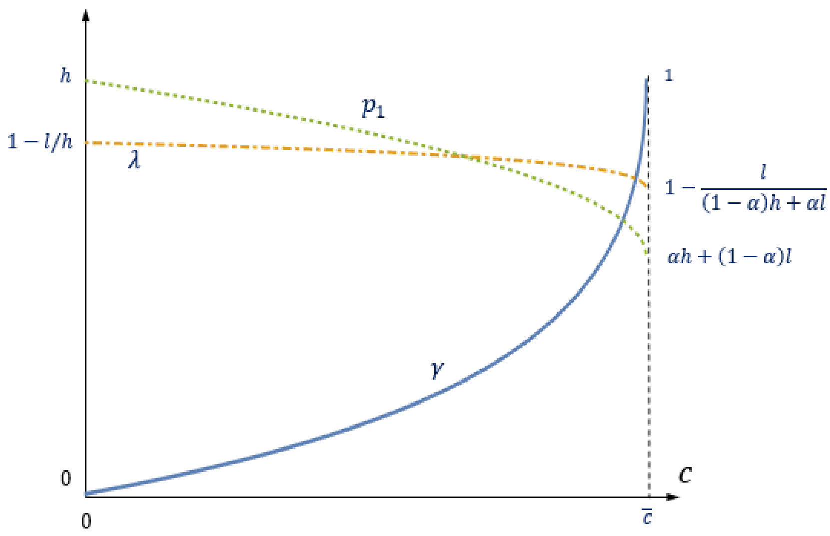

Figure 3. Equilibrium Strategies in the Mixed-strategy Equilibrium

Figure 3 illustrates equilibrium pricing strategies and customers' inspection decision. As can be seen from the figure, γ increases with the inspection cost, implying that, as the inspection cost increases, the firm tricks low-preference customers more frequently. This is because a higher inspection cost makes it more difficult for low-preference customers to discover their true preference, which gives the firm a stronger incentive to exploit them.

On the other hand, the high price p_1 decreases with c because, as the inspection cost increases, the firm tricks customers more often, raising customer suspicion about the firm's information abuse and reducing their willingness to pay when receiving a high price. Therefore, the firm must undercut its price to compensate for the increased inspection cost.

6. Discussion

Based on the analysis above, we found that, when $c < \bar{c} = \alpha(1-\alpha)(h-l)$, the equilibrium is a semi-separating equilibrium in which the firm randomizes its price offers to low-preference customers. When $c \geq \bar{c}$, however, the equilibrium is a pooling equilibrium in which the firm charges all customers their expected willingness to pay $\bar{v} = \alpha h + (1-\alpha)l$, and all customers make no-brainer purchases. In this section, we investigate how a firm's superior knowledge affects its profit, customer surplus, and social welfare, and discuss whether price discrimination and/or data collection should be regulated.

6.1. Firm Profit

First, we calculate the firm's equilibrium profit and come up with the following proposition.

***Proposition* 5**. *In equilibrium, the firm's profit is*

$$\pi = \begin{cases} \frac{\alpha}{2}\left(h + l + \sqrt{(h-l)(h-l-4c)}\right) + (1-\alpha)l & \text{if } c < \overline{c}, \\ \alpha h + (1-\alpha)l & \text{otherwise.} \end{cases}$$

The firm's profit decreases with the inspection cost when $c < \overline{c}$, jumps discontinuously at $c = \overline{c}$, and then becomes constant with c.

The intuition for Proposition 5 is as follows. When $c < \overline{c}$, the firm adopts a semi-separating equilibrium in which the firm's profit decreases with c because it bears indirect costs that accompany its superior knowledge of customer preferences. Customers take the firm's opportunistic incentive to trick low-preference customers into account and, upon receiving a high price, inspect with a positive probability. As such, the firm must distort the thought-provoking price p_1 down to compensate for customers' increased inspection cost, which works to the firm's detriment.

When $c \geq \overline{c}$, the inspection cost is so high that the firm cannot afford to trick customers and, as a result, forgoes its superior information and charges all customers a uniform price that equals their expected willingness to pay. Given the high inspection cost, all customers make no-brainer purchases and the firm's profit is maximized. The discontinuity at $c = \overline{c}$ showcases the firm's regime switch between the two strategies.

Then, does the firm actually benefit from its superior knowledge of customer preferences? To address this question, we compare the firm's profit with its profit under the no superior knowledge benchmark. The results are summarized in the following proposition.

Proposition 6. *The firm can be worse off with its superior knowledge when the inspection cost is moderate, i.e.,* $(1-\alpha+\alpha^2)\overline{c} < c < \overline{c}$.

Figure 4 illustrates the profitability of superior knowledge, where the dashed line represents the firm's profit under the no superior knowledge benchmark ($\hat{\pi}$), and the solid line represents the firm's profit with superior knowledge (π). When $c < (1-\alpha+\alpha^2)\overline{c}$, the firm is better off with superior knowledge since it can take advantage of the superior knowledge to better price-discriminate against customers. Even though the firm must distort the thought-provoking price down to compensate for customers' inspection cost, this effect is dominated by the boons of price discrimination.

The more interesting case lies within $(1-\alpha+\alpha^2)\overline{c} < c < \overline{c}$, the shaded area in Figure 4. Within this regime, the firm is worse off with superior knowledge because its opportunistic incentive to abuse its information advantage raises customer suspicion, which encourages inspection and forces the firm to distort its prices to compensate for customers' efforts. Within this range, the price distortion is so severe that it offsets the benefits of price-discriminating and leaves the firm, though more knowledgeable, ultimately worse off.

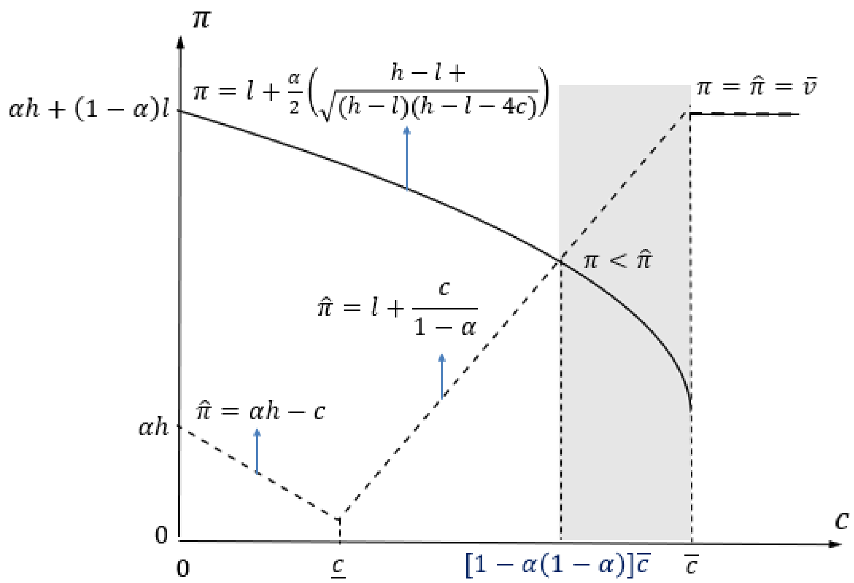

Figure 4. Profitability of Superior Knowledge

Finally, when $c > \overline{c}$, the inspection cost is so high that inspection is prohibitive for customers, and the firm always offers customers a uniform price regardless of whether or not it has superior knowledge. Therefore, superior knowledge has no effect on the firm's profit. These results defy conventional wisdom, which suggests that a price-discriminating firm always benefits from its superior knowledge (Xu and Dukes 2021) and suggest that superior knowledge may backfire on profits under customer inspection.

6.2. Customer Surplus and Social Welfare

We now assess the effect that pricing with superior knowledge has on customer surplus. First, consider the case in which $c < \overline{c}$: Within this regime, the firm offers two prices to customers, $p_1 > p_0 = l$. Only low-preference customers receive the low price p_0, and each enjoys a surplus of $CS_1 = \Omega(p_0)h + (1 - \Omega(\mathrm{p}_0))l - p_0 = 0$. For customers who receive a high price p_1, their expected surplus is

$$\Omega(p_1)h + (1 - \Omega(p_1))l - p_1 = -c + \Omega(p_1)(h - p_1) = 0.$$

In either case, customers break even. Second, consider the case in which $c \geq \overline{c}$: All customers receive the same price, equal to their expected willingness to pay. Customers break even again. Comparing these results to that of the no superior knowledge benchmark, we derive the following proposition.

Proposition 7. *Under price discrimination with superior knowledge, customer surplus is always zero. Relative to the no-superior-knowledge benchmark, personalized pricing harms customers on average when* $\underline{c} \leq c < \overline{c}$.

We have already shown that the firm cannot fully extract customer value when the inspection cost is low as it must inevitably distort its price p_1 down such that $p_1 < h$. Nonetheless, customers do not benefit from the firm's price discrimination because the gain from the price distortion is fully offset by the inspection cost that they incur.[10]

[10] In equilibrium, the expected payoff of a low-preference customer is negative while that of a high-preference customer is positive. Nonetheless, combining these two cases, customers' expected payoff is zero.

Figure 5 illustrates the effects of pricing with superior knowledge on the firm and its customers. Specifically, when α is large and the inspection cost is low (as depicted in Region I), superior knowledge benefits the firm without affecting customers. This is because superior knowledge allows the firm to expand its market to incorporate low-preference customers, who are uncovered without superior knowledge. This reflects the most highly touted benefit of price discrimination. When $\underline{c} < c < (1 - \alpha + \alpha^2)\overline{c}$ (as depicted in Region II), superior knowledge benefits the firm at the expense of customers. This is because such superior knowledge allows the firm to better appropriate customers with refined price-discrimination efforts and, because the inspection cost is low, the cost of customer suspicion is also low. When $(1 - \alpha + \alpha^2)\overline{c} < c < \overline{c}$ (as depicted in Region III), superior knowledge leads to a lose-lose outcome. On the one hand, the firm is worse off because the cost of customer suspicion offsets the benefit of price discriminating. On the other hand, customers are worse off because the firm appropriates customers through refined price discrimination. Finally, when $\overline{c} < c$ (as depicted in Region IV), the inspection cost is so high that the collection of customer data has no effects on the firm or its customers.

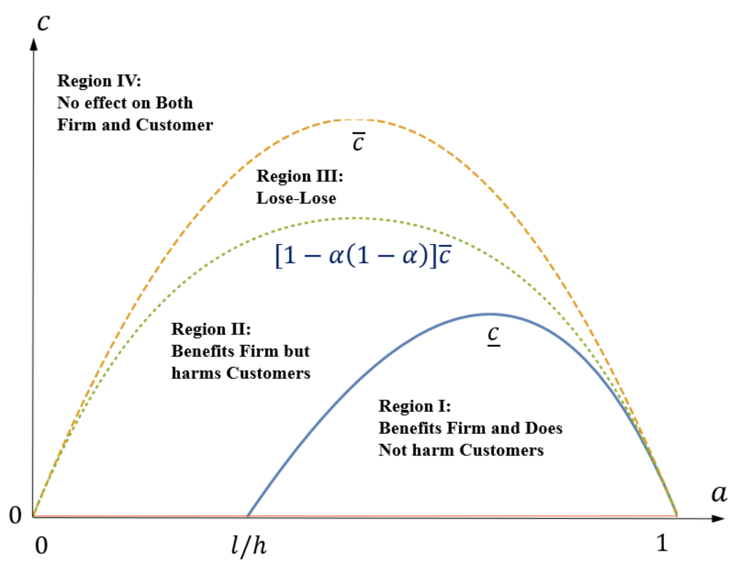

Figure 5. The Effects of Superior Knowledge on the Firm and Customers

The following proposition sheds light on total social welfare, which is the sum of customer surplus and firm profit.

Proposition 8. *Pricing with superior knowledge improves social welfare when* $c < \underline{c}$ *but can hurt social welfare when* $\underline{c} < c < \overline{c}$*. It has no effect on social welfare when* $c \geq \overline{c}$*.*

When $c < \underline{c}$, pricing with superior knowledge improves social welfare through market expansion: Without superior knowledge, the firm forgoes low-preference customers and focuses exclusively on gaining high-preference customers. Superior knowledge allows the firm to expand the market and obtain some low-preference customers who are otherwise uncovered. When $\underline{c} < c < \overline{c}$, superior knowledge can hurt social welfare because (1) a deadweight loss is incurred when customers inspect and, (2) upon receiving a high-price, some customers will inspect and reject purchase when they find out their low match value, thus reducing market transactions. Both effects lead to a social loss.

6.3. Implications for Public Policy

Economists and public policymakers often adopt positive takes on the use of information technologies and data analytics for price discrimination. Such strategies not only expand "the size of the market by charging more to those willing to pay and less to those who are not" (Furman and Simcoe 2015) but also help firms better meet their customers' needs, interests, and priorities (Oxera 2017). This is why public policymakers often support certain degrees of data collection and price discrimination. In the United States, any form of price discrimination is legal so long as the basis of discrimination is not race, religion, national origin, gender, and the like (Ramasastry 2005). Nevertheless, customer advocates often worry that data analytics and price discrimination "transfer value from customers to shareholders, which generally leads to an increase in inequality and can therefore be inefficient from a utilitarian standpoint" (Furman and Simcoe 2015). As such, ongoing debates probe whether or not data collection and price discrimination should be regulated by policymakers through measures such as increasing data protection or prohibiting this practice altogether (Bar-Gill 2019).

We find that, consistent with common wisdom, price discrimination with superior knowledge does expand the market and improve social efficiency when the inspection cost is low. However, this is not necessarily true when the inspection cost is moderate, as exemplified by Proposition 8. Within this regime, superior knowledge can be used by firms to set severely opportunistic prices, which raises excessive customer suspicion. This reduces the market size by deterring some low-preference customers and hurts firm profits.

Our results inform public policymakers that price discrimination with superior knowledge can lead to unintended consequences for firms, customers, and social welfare alike and, thus, taking a stronger stance against its overuse (e.g., banning) may lead to a win-win outcome. Our results also suggest that firms may not always take a negative view of the restrictions placed on data collection used for price discriminating. Instead, they should support these restrictions when the inspection cost is moderate as a way to protect their profits.

It is worthwhile to mention that, when price discrimination is legal and customers do not observe a firm's information acquisition efforts (which is typical practice as presented in Li et al. 2020), the firm will have a commitment problem. In other words, the firm would be better off if it can commit to not colleting and using superior knowledge when setting its prices but, without this commitment power, it cannot help but to secretly collect superior knowledge and use it for price discrimination; Fudenberg and Villas-Boas (2006) and Li et al. (2020) offered detailed discussions on this issue. Given that firms cannot refrain from price discriminating with superior knowledge, public policy is required to restore market efficiency.

7. Model Generalizations

The basic model studies a scenario in which the firm has perfect information about customers' match values, and offers customers personalized prices accordingly. In this section, we extend the basic model in two directions. In Section 7.1, we allow the firm's superior information to be imperfect by assuming that the firm receives noisy signals about customer preferences. In Section 7.2, we incorporate the design of product quality into the firm's strategy space, which helps expand the scope of our analysis beyond personalized pricing framework.

7.1. Firm's Information is Imperfect

The basic model assumes that the firm's superior knowledge is perfect, i.e., the firm identifies the customers' match values perfectly. However, in practice, the firm's information is often noisy and imperfect due to insufficient behavioral data or imperfect data-analytics algorithms. To capture this scenario, we extend the basic model by studying the equilibrium outcome when the firm's superior knowledge is imperfect.

Consider the following model. The firm receives a signal $s_i \in \{\tilde{h}, \tilde{l}\}$ about the match value of customer i that follows the following distribution:

$$\Pr[\tilde{h}|h] = \Pr[\tilde{l}|l] = \beta, \Pr[\tilde{l}|h] = \Pr[\tilde{h}|l] = 1 - \beta.$$

where $\beta \in [1/2, 1]$ captures the reliability of the firm's superior knowledge, and a higher value of β implies that the firm's superior knowledge is more reliable. In the extreme case of $\beta = 1/2$, the firm's superior knowledge is completely unreliable, i.e., it has no improvement over the prior; in the extreme case of $\beta = 1$, the firm's superior knowledge is perfect, i.e., it reveals customer types with certainty. Figure 6 illustrates the information structure of the game.

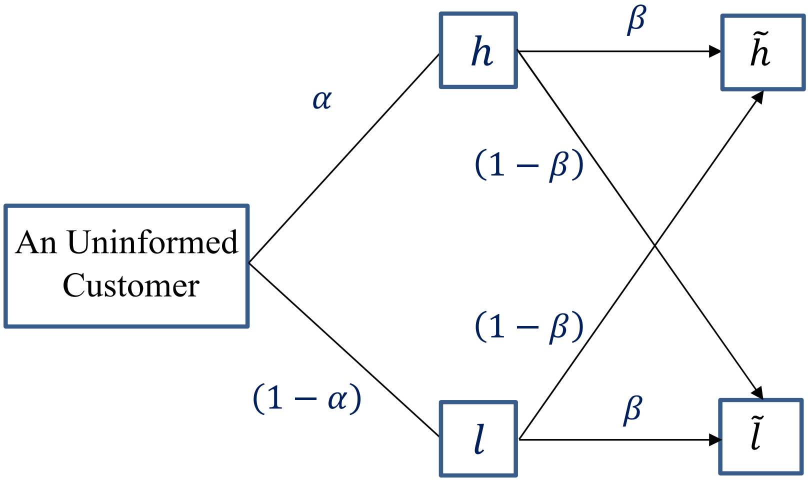

Figure 6: Information Structure of the Game

In the basic model, we show that when the inspection cost is not excessively high, a mixed-strategy equilibrium exists in which the firm randomizes its pricing offers to low-preference customers. In this extension, we show that a similar semi-separating equilibrium exists when customers' inspection cost is moderate. Details of the analysis is relegated to Appendix B.

Proposition 9. *For intermediary regions of inspection cost* $c \in \left(\frac{1-e_H}{2-e_H}(e_H h - l), \bar{c}\right]$*, there exists a semi-separating equilibrium with two prices* p_1 *and* p_0*, where*

$$p_1 = \frac{h+l+\sqrt{(h-l)(h-l-4c)}}{2},\ p_0 = \frac{\alpha(1-\beta)h+(1-\alpha)\beta l}{\alpha(1-\beta)+(1-\alpha)\beta}, e_H \equiv \Pr[h|\tilde{h}] = \frac{\alpha\beta}{\alpha\beta+(1-\alpha)(1-\beta)}.$$

In equilibrium, the firm offers all high-signal customers (i.e., customers with a signal $\tilde{h}$*) price* p_1 *but randomizes its price offers to low-signal customers: With probability* γ*, it offers low-signal customers price* p_1*, and with probability* $1-\gamma$*, it offers them price* p_0*, where*

$$\gamma = 1 - \frac{2c(\alpha+\beta-2\alpha\beta)-(h-l)(1-\alpha)\alpha+(1-\alpha)\alpha\,(2\beta-1)\sqrt{(h-l)(h-l-4c)}}{2c\,(\alpha+\beta-2\alpha\beta)^2-2\,(h-l)(1-\alpha)\alpha(1-\beta)\beta}.$$

Upon observing price p_1*, a customer randomizes her inspection decision. With probability* λ*, she inspects the product and makes a purchase if and only if her match value is high. With probability* $1-\lambda$*, she makes a no-brainer purchase. Upon observing the price* p_0*, she always makes a no-brainer purchase. The inspection probability* λ *upon observing* p_1 *is given by*

$$\lambda = \frac{(\alpha+\beta-2\alpha\beta)\sqrt{(h-l)(h-l-4c)}-(h-l)(\alpha-\beta)}{(1-\alpha)\beta\left(h+l+\sqrt{(h-l)(h-l-4c)}\right)}.$$

Note that Proposition 4 is a special case of Proposition 9 when $\beta = 1$ and $e_H = 1$. It is worth noting that when $c < \frac{1-e_H}{2-e_H}(e_H h - l)$, there may exist a pooling equilibrium in which the firm offers a uniform price to all customers.

Consider the following numerical example: $\alpha = 0.5, \beta = 0.8, c = 0.15, h = 1$ and $l = 0.2$. In equilibrium, the firm will always offer a high-signal customer $p_1 \approx 0.8828$ and randomize its price offer to a low-signal customer. With probability $\gamma \approx 9.1\%$, the firm tricks the customer by overcharging her p_1, while with probability $1 - \gamma \approx 90.9\%$, the firm offers the customer $p_0 = 0.36$. Upon receiving the low price p_0, the customer always makes a no-brainer purchase; however, when she receives the price p_1, she inspects the product with probability $\lambda = 68.75\%$ and makes a no-brainer purchase otherwise.

For the impact of imperfect knowledge on firm profit, we present the following proposition.

Proposition 10. *Under the semi-separating equilibrium, the firm's profit strictly decreases with β, the reliability of the signal.*

Proposition 10 suggests that the firm may prefer imperfect knowledge to perfect knowledge because as the firm's signal becomes unreliable, the likelihood of false positives and false negatives increases, and the relative difference between high-signal and low-signal customers decreases. In this case outcome, the firm's incentive to trick customers also declines. In anticipation of this, customers inspect less often in equilibrium, and are more willing to make a no-brainer purchase upon observing a high price, which saves on the deadweight loss of inspection and improves the firm's profit.

7.2. Managing Customer Suspicion: Optimal Quality Decision

We found that, when pricing with superior knowledge, the firm often has opportunistic incentives to abuse its information advantage and trick low-preference customers into overpaying for its product, which, in turn, raises customer suspicion and undermines pricing efficiency. In this section, we show that the firm can curb its opportunism and alleviate customer suspicion by making optimal decisions regarding product quality, assuming that price discrimination can be legally utilized.

Our basic model assumes an exogenous product quality to focus on the firm's pricing decision. In this section, we extend the model by letting the firm decide both the quality of its product and its prices. Note that products of higher quality amplify valuation uncertainty and raise the need for customer inspection. As such, we investigate whether or not the firm can curb its opportunism and alleviate customer suspicion through its product quality.

To examine these issues, we extend the main model to incorporate the firm's quality decision. Here, we define quality in the vertical sense (Moorthy 1984; Hu et al. 2015). First, we assume that the firm chooses from a closed interval $[0, 1]$ of feasible levels of product quality. The upper bound, $\hat{q} = 1$, represents the limits of the firm's production technology. The firm can choose to produce a full-quality product $q = \hat{q}$ or an inferior product at any point of the interval. Second, at the point of purchase, customers know the quality but are uncertain about their marginal valuation for this quality; see Guo and Zhang (2012) and Li et al. (2019) for similar assumptions. We assume that customers' true valuations, measured by their marginal willingness to pay for a product's quality, are heterogeneous. Specifically, for a quality level q, the product valuation is $v_i = \theta_i q$, where θ_i, customer i's marginal willingness to pay for quality, follows the following two-point distribution:

$$\theta_i = \begin{cases} h & \text{with probability } \alpha, \\ l & \text{with probability } 1 - \alpha. \end{cases}$$

Multiplicative formulation is standard in the literature on vertical differentiation (Moorthy 1984, Li et al. 2019). The basic model is, therefore, a special case in which the firm offers a full-quality product, i.e., $q = 1$. Third, as in the main model, customers do not know but can find out the true value of θ_i at an inspection cost c. The firm, by contrast, can assess θ_i for each individual customer. Finally, to focus on the strategic effect that customer suspicion has on the firm's quality decision, we let the marginal production cost for quality equal zero. We allow this to show that, even when the concern for production costs is absent, the firm may not choose the highest quality for its product as not to raise customer suspicion. Our results hold when a high-quality product is more costly to produce.

We add a stage 0 to the basic model in which the firm chooses the product quality, $q \in [0,1]$. The remainder of the game is unchanged. The following proposition speaks to the firm's optimal quality decisions.

Proposition 11. *There exists $c_0 < \bar{c}$ such that the firm strictly prefers an inferior product when $c_0 < c < \bar{c}$.*

Figure 7 illustrates the firm's optimal product quality. As can be seen, with moderate inspection costs, the firm offers an inferior quality product, as depicted by the shaded area, even if a higher quality product is no more costly to produce.

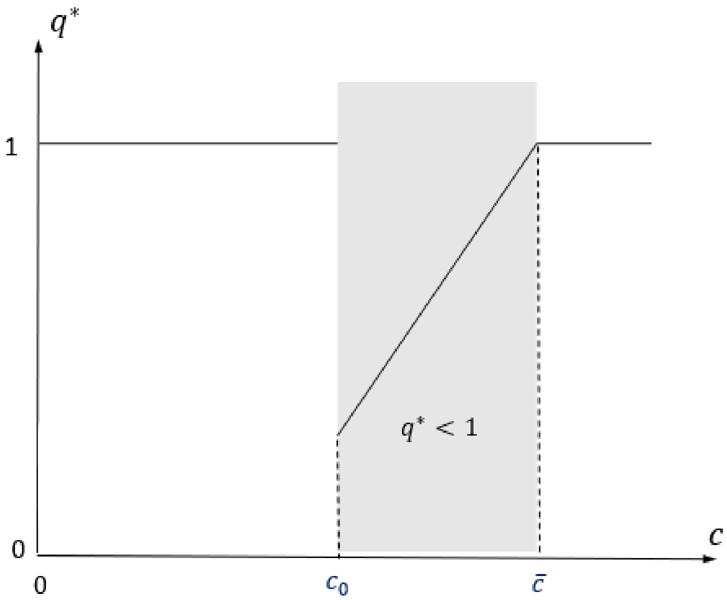

Figure 7. The Optimal Product Quality with the Inspection Cost

But why would the firm prefer to offer an inferior product? The intuition is as follows. When quality is lowered, customer valuation is lowered along with the customers' benefits in finding out their preferences. In other words, the "quality-adjusted inspection cost," $\frac{c}{q}$, increases as q decreases. When this quality-adjusted inspection cost is high enough, customers never inspect regardless of the price the firm offers, and the resulting equilibrium will be the single-price pooling equilibrium that prevents the firm from acting opportunistically. Thus, quality distortion reduces customer value as well as eliminates customer suspicion. Within a certain range, the benefit of eliminating customer suspicion dominates the cost of decreasing customer value, and the firm is better off offering an inferior product.

Interestingly, because customer surplus is always zero, social welfare is always equal to firm profits; therefore, we conclude that offering an inferior product can also improve social efficiency. Even though the inferior product decreases customer value, it eliminates the inspection cost, a deadweight loss that would otherwise be incurred by customers.

8. Concluding Remarks

In many markets such as credence goods and dating services, a firm knows both its product characteristics and customers' needs, and can accurately assess their match values for its product and tailor its price offers accordingly. Customers, on the other hand, may not know their own preference for the firm's product, and must invest in costly inspection efforts to find out if they are subject to exploitation.

This paper investigated a scenario in which a firm has superior knowledge about customer preferences, which customers themselves must find out through costly inspection efforts. We developed and analyzed a model in which the firm, using its superior knowledge of customer preferences, price-discriminates against customers who may invest in inspection to find out their true valuation of a product and willingness to pay a certain price. We find that customer inspection has substantial effects on the firm's equilibrium pricing strategy. To review our main findings, we return to the research questions we established at the start of this work.

How should a firm take advantage of its superior knowledge to price discriminate against customers? Upon observing a personalized price, will the customer trust the firm and make a no-brainer purchase of its product?

On the surface, it seems that a firm can fully exercise its information advantage to implement first-degree price discrimination against its customers and reap the entire customer value. Our analysis reveals, however, that this assumption does not always hold. With a positive inspection cost, the firm always has an opportunistic incentive to trick low-preference customers into overpaying for its product, which raises suspicions among savvy customers. This suspicion leads them to incur an inspection cost that would be avoided if the firm were not capable of personalized pricing. Though the firm could simply stick with uninform pricing, such a pooling equilibrium does not survive the D1 criterion unless the inspection cost is sufficiently high. As a result, the firm must distort its prices to compensate for inspection efforts and thus cannot fully extract customer surpluses. In equilibrium, when the inspection cost is not too high, the firm uses a semi-separating equilibrium: It always charges high-preference customers a high price but randomizes its price offers to low-preference customers, i.e., the firm sometimes charges an honest, low price but, at other times, tricks low-preference customers into overpaying a higher price. Faced with the high price, customers remain uncertain about their actual preference and inspect with positive probabilities.

How does the cost of inspection affect the firm and its customers? Should policymakers regulate data collection and price discrimination to protect customers and social welfare?

We compared equilibrium outcomes with and without the firms' superior knowledge and price discrimination and found that, when the inspection cost is low, the firm does benefit from its superior knowledge. When the inspection cost is moderate, however, customer suspicion is too severe that the firm has to distort its price significantly to compensate for customers' inspection costs, and such a distorted price leaves the firm worse off. In addition, the ex-ante customer surplus can also be (weakly) worse off when the firm uses price discrimination, which implies that this strategy can lead to a "lose-lose" situation. Our findings defy the belief that superior information always benefits a firm and that price discrimination improves social efficiency through increased market coverage. Our findings also suggest that banning data collection and price discrimination may not only benefit customers but also firms and society as a whole. As such, public policymakers should take a stronger stance against data collection and price discrimination.

How can a firm manage customer inspections and improve profits through decisions other than prices?

Firms may want to curb their opportunistic tendency to alleviate customer suspicion and improve profits. We extended our model by allowing the firm to choose the quality of its product and found that, even if a higher-quality product is costless to produce, the firm may intentionally select a product of inferior quality. By offering a low-quality product, the quality-adjusted inspection cost increases, thus preventing customers from inspecting and ultimately benefiting the firm.

Our results, of course, are not the final say on price discrimination with superior knowledge. Instead, we see this concept and practice as an exciting new avenue for future research and believe our research can be extended in several directions. First, the current model only examines situations with a monopolist firm. Future studies may analyze an oligopolistic market in which multiple firms collect customer data to obtain superior knowledge and tailor offers. It may also be interesting to examine the effect of price discrimination under customer inspection in an anti-trust framework. Second, our stylized model focuses on the use of superior knowledge for price discrimination. In practice, firms commonly use superior knowledge to make non-price decisions such as recommending specific products to certain customers or garnering prospective customers with targeted ads. Future works may examine other implications of using superior knowledge to communicate with uninformed customers via innovative mechanisms. Third, customer attitudes towards price discrimination can significantly affect the spread of its practice. In our model, we assume that all customers are rational and risk neutral. But, in reality, behavioral factors such as concerns for distributional as well as peer-induced fairness may arise within the strategic interactions between a data-collecting firm and its inspecting customers as well as among customers themselves (Guo and Jiang 2016; Li and Jain 2016). Therefore, we view our research only as a rational cross-section of complex processes that involve both behavioral attitudes and rational information communication.

References

Acquisti, A. and Varian, H.R., 2005. Conditioning prices on purchase history. *Marketing Science*, 24(3), 367–381.

Banks, J.S. and Sobel, J., 1987. Equilibrium selection in signaling games. *Econometrica*, 647-661.

Bar-Gill, O. 2019. Algorithmic price discrimination: When demand is a function of both preferences and (mis)perceptions. *University of Chicago Law Review,* 86(2), article 12.

Cao, X., and Zhang, J. 2021. Preference learning and demand forecast. *Marketing Science*, 40(1), 62-79.

Chen, Y. and Iyer, G., 2002. Consumer addressability and customized pricing. *Marketing Science*, 21(2), 197-208.

Chen, Y., Li, X. and Sun, M., 2017. Competitive mobile geo targeting. *Marketing Science*, 36(5), 66–682.

Chen, Y., Narasimhan, C. and Zhang, Z.J., 2001. Individual marketing with imperfect targetability. *Marketing Science*, 20(1), 23–41.

Chen, Y. and Zhang, Z.J., 2009. Dynamic targeted pricing with strategic consumers. *International Journal of Industrial Organization*, 27(1), 43–50.

Cho, I.K. and Kreps, D.M., 1987. Signaling games and stable equilibria. *Quarterly Journal of Economics*, 102(2), 179-221.

Fong, Y.F., 2005. When do experts cheat and whom do they target? *RAND Journal of Economics*, 36(1), 113-130.

Fudenberg, D. and Tirole, J. 2000. Consumer poaching and brand switching. *RAND Journal of Economics*, 31(4), 634-657.

Fudenberg, D. and Villas-Boas, J.M., 2006. Behavior-based price discrimination and consumer recognition. *Handbook on economics and information systems*, 1, 377–436.

Furman, J., and Simcoe, T. 2015. *The economics of big data and differential pricing*. The White House President Barack Obama.

Green, A. 2018. Google knows you better than you know yourself. *Komando* (April 1).

Guo, L. 2016. Contextual deliberation and preference construction. *Management Science*, 62(10), 2977-2993.

Guo, L. and Wu, Y., 2016. Consumer deliberation and quality signaling. *Quantitative Marketing Economics*, 14, 233–269.

Guo, L., and Zhang, J. 2012. Consumer deliberation and product line design. *Marketing Science*, 31(6), 995-1007.

Guo, X. and Jiang, B., 2016. Signaling through price and quality to consumers with fairness concerns. *Journal of Marketing Research*, 53(6), 988-1000.

Hajihashemi, B., Sayedi, A., and Shulman, J. D. 2020. The perils of personalized pricing with network effects. Working paper, University of Washington.

Heilweil, R. 2020. Tinder may not get you a date. It will get your data. *Vox* (Feb 14).

Hu, M., Li, X. and Shi, M., 2015. Product and pricing decisions in crowdfunding. *Marketing Science*, 34(3), 331-345.

Jiang, B., Ni, J. and Srinivasan, K., 2014. Signaling through pricing by service providers with social preferences. *Marketing Science*, 33(5), 641-654.

Jing, B., 2011. Seller honesty and product line pricing. *Quantitative Marketing and Economics*, 9(4), 403-427.

Jing, B. 2016. Consumer recognition in experience vs. inspection good markets. *Management Science*, 62(1), 216-224.

Jones, A. L. (2017). The gluten-free diet: fad or necessity? *Diabetes Spectrum*, 30(2), 118-123.

Koksal, I. 2018. Artificial intelligence may know you better than you know yourself. *Forbes* (Feb 27).

Li, K. J. and Jain, S. 2016. Behavior-based pricing: an analysis of the impact of peer-induced fairness. *Management Science*, 62(9), 2705-2721.

Li, X., Li, K. J., and Wang, X. 2020. Transparency of behavior-based pricing. *Journal of Marketing Research*, 57(1), 78-99.

Li, X., Li, Y. and Shi, M., 2019. Managing consumer deliberations in a decentralized distribution channel. *Marketing Science*, 38(1), 170–190.

Mattioli, D. 2012. On Orbitz, Mac users steered to pricier hotels. *Wall Street Journal* (August 23).

Moorthy, K.S., 1984. Market segmentation, self-selection, and product line design. *Marketing Science*, 3(4), 288-307.

Oxera Consulting LLP. 2017. When algorithms set prices: winners and losers. *Discussion paper*.

Pang, S. H. (2018). Critical issues in the air force medical equipment procurement process. Wright-Patterson AFB, OH: Air Force Institute of Technology.

Perez, S. 2019. Target's personalized loyalty program launches nationwide next month. *TechCrunch* (September 10).

Ramasastry, A. 2005. Websites that charge different consumers different prices: Is their 'price customization' illegal? should it be? *FindLaw* (June 20).

Research and Markets. 2020. Global Big Data Analytics Market Size, Market Share, Application Analysis, Regional Outlook, Growth Trends, Key Players, Competitive Strategies and Forecasts, 2019 To 2027, Report.

Rhee, K.E. and Thomadsen, R., 2017. Behavior-based pricing in vertically differentiated industries. *Management Science*, 63(8), 2729–2740.

Rijmenam, M. V. 2017. How Artificial Intelligence Will Change the Travel Industry. ODSC Conference (November 30).

Roettgers, J. 2018. Netflix's Secrets to Success: Six Cell Towers, Dubbing and More. *Variety* (March 8).

Shin, J. and Sudhir, K. 2010. A customer management dilemma: when is it profitable to reward one's own customers? *Marketing Science*, 29(4), 671-689.

Shin, J., Sudhir, K. and Yoon, D.H., 2012. When to "fire" customers: Customer cost-based pricing. *Management Science*, 58(5), 932-947.

Shugan, S. M. 1980. The cost of thinking. *Journal of Consumer Research*, 7(2), 99-111.

Stephens-Davidowitz, S. Everybody lies: Big data, new data, and what the Internet can tell us about who we really are. *Harper Collins*, 2017.

Stigler, G.J. 1966. *The theory of price*, New York: Macmillan.

Thisse, J. F., and Vives, X. 1988. On the strategic choice of spatial price policy. *American Economic Review*, 122-137.

Wathieu, L., and Bertini, M. 2007. Price as a stimulus to think: The case for willful overpricing. *Marketing Science*, 26(1), 118-129.

Wolinsky, A., 1993. Competition in a market for informed experts' services. *RAND Journal of Economics*, 24(3), 380-398.

Xu, Z. and Dukes, A., 2019. Product line design under preference uncertainty using aggregate consumer data. *Marketing Science*, 38(4), 669–689.

Xu, Z. and Dukes, A., 2021. Personalization from customer data aggregation using list price. *Management Science*, https://doi.org/10.1287/mnsc.2021.3977.

Appendix

A. Equilibrium Characterization

A.1. Analysis of Semi-Separating Equilibrium.

We now characterize the semi-separating equilibrium in which the firm sometimes offers the same price to both customer types but, at other times, offers different prices to the two customer types.

To facilitate our analysis, we first introduce the term "thought-provoking price".

Definition 1. A price p is called a thought-provoking price if and only if at this price, customers inspect with positive probabilities.

In other words, at a thought-provoking price, at least some customers are willing to inspect. We next prove the following auxiliary lemma to facilitate our analysis.

Lemma A1. *At any thought-provoking price, customers either inspect (and make purchases when their types are high) or make no-brainer purchases.*

Proof of Lemma A1. Assume for contradiction that there exists a thought-provoking price p such that $p > \Omega(p)h + \big(1 - \Omega(p)\big)l$. Consider a low-preference customer. Then, faced with price p, the customer either makes a no-brainer no-purchase, or inspects and rejects purchase. In either case, the low-preference customer does not purchase. This is clearly suboptimal for the firm because it can charge $p = l$ to the low-preference customer and make a positive profit from her. Following this logic, this price can only be offered to high-preference customers. However, given this price is only offered to high-preference customers, we must have that $\Omega(p) = 1$, and no customers will inspect upon receiving price p. This contradicts the assumption that p is a thought-provoking price.

According to the analysis above, we know that all thought-provoking price must satisfy that $p \leq \Omega(p)h + \big(1 - \Omega(p)\big)l$. With this property, a customer will either inspect or make a no-brainer purchase, which proves the lemma. **Q.E.D.**

Next, we prove the second auxiliary lemma.

Lemma A2. *In any equilibrium, there is at most one thought-provoking price.*

Proof of Lemma A2. Note first that a though-provoking price is charged to at least some of high-preference customers. Otherwise, customers know that they are low-preference customers when receiving the price and will not inspect, which contradicts the definition of thought-provoking price.

Now, assume for contradiction that there exist two thought-provoking prices p_1 and p_2 such that $p_1 > p_2$. Then, following Lemma A1, a high-preference customer will always make a purchase at these prices (regardless of whether or not she inspects). Suppose that the firm offers price p_2 to a high-preference customer i, then the firm will make a profit p_2 from this customer. However, if the firm deviates and offers this customer price p_1, the customer will still make a purchase and the firm can make a profit $p_1 > p_2$ from her. Therefore, the firm will never offer price p_2 to a high-preference customer, a contradiction. **Q.E.D.**

Finally, we prove the third auxiliary lemma.

Lemma A3. *In any equilibrium, there exists at most one price at which all customers make no-brainer purchases.*

Proof of Lemma A3. Assume for contradiction that there exist two equilibrium prices p_1 and p_2 at which all customers are willing to make no-brainer purchases, $p_1 > p_2$. Consider a customer who receives price p_2. Clearly, the firm can improve its profit by offering this customer price p_1 and still having her make a no-brainer purchase, a contradiction. **Q.E.D.**

Now we move on to analyze the semi-separating equilibrium. With the above three auxiliary lemmas, we can show immediately that in a semi-separating equilibrium, there exist exactly two unique prices: a thought-provoking price p_1 and an inspection-prevention price p_0 at which customers make no-brainer purchases.

Moreover, the prices satisfy that $p_1 > p_0$. Otherwise, the firm can improve profit by offering all customers price p_0. In equilibrium, all high-preference customers must be offered the thought-provoking price p_1. This is because when offered this price, high-preference customers will either inspect or make no-brainer purchases; in either case they will end up purchasing the product. Therefore, the firm has no incentive to offer them the low price p_0.

With the above analysis, we know that in equilibrium, all high-preference customers are offered price p_1, while some low-preference customers are offered price p_1 and other low-preference customers are offered price p_0. Let γ denote the fraction of low-preference customers that receive the thought-provoking price p_1. Applying the Bayes' rule, we obtain the posterior beliefs as follows.

$$\Omega(p_1) = \frac{\alpha}{\alpha + (1-\alpha)\gamma}, \qquad \Omega(p_0) = 0.$$

Suppose that given the thought-provoking price p_1, a customer inspects with probability λ. We now characterize the key parameters p_1, p_0, γ and λ.

First, the firm's willingness to randomize implies that it must be indifferent about offering a low-preference customer price p_1 and p_0. This leads to

$$(1-\lambda)p_1 = p_0.$$

Second, customers' willingness to make no-brainer purchases at p_0 implies that

$$\Omega(p_0)h + \big(1-\Omega(p_0)\big)l \geq p_0.$$

Third, customers' willingness to make no-brainer purchases at p_1 implies that

$$\Omega(p_1)h + \big(1-\Omega(p_1)\big)l \geq p_1.$$

Fourth, at the thought-provoking price, a customer must be indifferent about whether to inspect. This yields that

$$\Omega(p_1)h + \big(1-\Omega(p_1)\big)l - p_1 = -c + \Omega(p_1)(h-p_1).$$

The firm's equilibrium profit is $\pi = \alpha p_1 + (1-\alpha)p_0$. This is because all high-preference customers receive price p_1 and make purchases; as for low-preference customers, the firm's willingness to randomize implies the firm gets a profit of p_0 from each of them. Solving for the equilibrium strategies we obtain the followings:

$$p_1 = \frac{h + l + \sqrt{(h-l)(h-l-4c)}}{2}, \qquad p_0 = l,$$

and

$$\lambda = \frac{\alpha\left(h - l - 2c - \sqrt{(h-l)(h-l-4c)}\right)}{2(1-\alpha)c}.$$

Note that in the semi-separating equilibrium, we must have that $0 \leq \lambda \leq 1$. This is translated into $c \leq \alpha(1-\alpha)(h-l)$. Otherwise, when the inspection cost is large, no semi-separating equilibrium exists.

A.2. Analysis of Pooling Equilibrium.

In a pooling equilibrium, the firm offers a single price p to all customers. Because the equilibrium is pooling, customers' belief must be $\Omega(p) = \alpha$. Following Lemmas A2 and A3, we know that in a single-price equilibrium, the firm either offers a price that induces no-brainer purchases or offers a though-provoking price. Consider first the case in which the firm offers a price that induces no-brainer purchases.

The individual rationality constraint states that customers must be willing to make a purchase:

$$\alpha h + (1-\alpha)l \geq p.$$

The incentive compatibility constraint says that customers must not be willing to inspect:

$$\alpha h + (1-\alpha)l - p \geq \alpha(h-p) - c.$$

The firm's profit in this case is $\pi = p$. Simple algebra suggests that the firm optimally charges the following price:

$$p = \begin{cases} l + \dfrac{c}{1-\alpha}, & \text{if } \; c < \alpha(1-\alpha)(h-l), \\ \alpha h + (1-\alpha)l, & \text{otherwise.} \end{cases}$$

Next, suppose that the firm charges a thought-provoking price. According to Lemma A1, customers must be willing to make no-brainer purchases at this price, i.e.,

$$\alpha h + (1-\alpha)l \geq p.$$

Moreover, according to the definition of thought-provoking price, customers must be indifferent about whether or not to inspect, which suggests that

$$\alpha h + (1-\alpha)l - p = \alpha(h-p) - c.$$

Simple calculation shows that such an equilibrium, if existent, is dominated by the pooling equilibrium described above.

A.3. Equilibrium Refinement.

Following the discussion above, we know that when $c \leq \bar{c} = \alpha(1-\alpha)(h-l)$, there exist a two-price semi-separating equilibrium and a single-price pooling equilibrium. We now prove that the single-price pooling equilibrium fails the divinity criterion D1.

Suppose that there exists an equilibrium in which the firm offers both types of customers a single price $p^* = l + \frac{c}{1-\alpha}$. Now, consider an out-of-equilibrium price $p > p^* > l$. Let $\omega = \Pr(v_i = h|p)$ be the customers' posterior out-of-equilibrium belief upon observing p.

For low-preference customers, the firm weakly prefers to deviate to p if and only if they make no-brainer purchases when receiving price p, which translates to $\omega \in \Omega_L$, where

$$\Omega_L = \left[\max\left(1 - \frac{c}{p-l}, \frac{p-l}{h-l}\right), 1\right].$$

Note that given $\omega \in \Omega_L$, a customer will make a no-brainer purchase, and the firm's profit from that customer is $p > p^*$.

Consider next a high-preference customer. The firm strictly prefers to deviate to p when $\omega \in \Omega_H$, where

$$\Omega_H = \left(\frac{c}{h-p}, 1\right].$$

Note that given $\omega \in \Omega_H$, the customer either makes a no-brainer purchase, or inspects and makes an informed purchase (because the customer's type is high). In either case the firm's profit is $p > p^*$.

Now let $p = \frac{h+l+\sqrt{(h-l)(h-l-4c)}}{2} - \epsilon$ for some small positive $\epsilon > 0$. It can be shown that whenever $c < \alpha(1-\alpha)(h-l)$, we have (1) $p > p^*$ and (2) $\Omega_L \subsetneq \Omega_H$. Then, according to D1, customers' belief must be $\Omega(p) = 1$. However, given $\Omega(p) = 1$, the firm strictly prefers to offer high-preference customers price p instead of p^*, a contradiction. Therefore, the single-price pooling equilibrium fails the D1 criterion whenever $c < \alpha(1-\alpha)(H-L)$.

B. Proofs of Lemmas and Propositions

Proof of Lemma 1. Customers make no-brainer purchases whenever $CS_N \geq \max(CS_I, 0)$, which translates to $p \leq \min\left(l + \frac{c}{1-\alpha}, \alpha h + (1-\alpha)l\right)$. Customers make no-brainer no-purchases whenever $CS_I < 0, CS_N < 0$, which translates to $\max\left(h - \frac{c}{\alpha}, \alpha h + (1-\alpha)l\right) < p$. Finally, customers inspect the product in other cases, i.e., when $l + \frac{c}{1-\alpha} < p \leq h - \frac{c}{\alpha}$. **Q.E.D.**

Proof of Proposition 1: Following the earlier discussions, the firm's optimization problem is to set a uniform price p to maximize $\pi = \alpha \times p$, subject to $l + \frac{c}{1-\alpha} < p \le h - \frac{c}{\alpha}$, or to maximize $\pi = p$, subject to $p \le \min\left(l + \frac{c}{1-\alpha}, \alpha h + (1-\alpha)l\right)$. By Lemma 1, it suffices to discuss two situations:

(1) If $l + \frac{c}{1-\alpha} < h - \frac{c}{\alpha}$, or equivalently, $c < \bar{c} = \alpha(1-\alpha)(h-l)$, then $\min\left(l + \frac{c}{1-\alpha}, \alpha h + (1-\alpha)l\right) = l + \frac{c}{1-\alpha}$. The firm needs to compare the maximum profits from either strategy: $p^* = h - \frac{c}{\alpha}$ and $p^* = l + \frac{c}{1-\alpha}$. Since $\alpha\left(h - \frac{c}{\alpha}\right) > l + \frac{c}{1-\alpha}$ is equivalent to $c < \underline{c} = \frac{(1-\alpha)(\alpha h - l)}{2-\alpha}$, we must have $p^* = h - \frac{c}{\alpha}$ if $c < \underline{c}$, and $p^* = l + \frac{c}{1-\alpha}$ if $\underline{c} \le c < \bar{c}$. The profits are then $\alpha h - \text{c}$ and $l + \frac{c}{1-\alpha}$, respectively. The total customer surplus is zero when $p^* = h - \frac{c}{\alpha}$, because $CS_I = 0$. And when $p^* = l + \frac{c}{1-\alpha}$, $CS_U = \alpha h + (1-\alpha)l - p^* = \alpha(h-l) - \frac{c}{1-\alpha}$.

(2) If $l + \frac{c}{1-\alpha} \ge h - \frac{c}{\alpha}$, which is equivalent to $c \ge \bar{c}$, then by Lemma 1, customers never inspect since $CS_I < \max(CS_N, 0)$. In this case, the firm can charge a price up to $CS_N = 0$, which implies that $p^* = \alpha h + (1-\alpha)l$. Since all customers purchase, the profit is also $\alpha h + (1-\alpha)l$ and the entire ex-ante customer surplus is exploited. **Q.E.D.**

Proof of Lemma 2: The argument is straightforward from the analysis in Section 5.1. Assume for contradiction that for some $c > 0$ there exists an equilibrium in which the firm implements first-degree price discrimination. Given $p_H = h$ and $p_L = l$, customers learn their values from the prices directly and would make no-brainer purchases whenever $c > 0$. But if both types of customers make a no-brainer purchase, then the firm has incentives to deviate by overcharging the L-types, having them pay $p_H = h$ instead. A contradiction. **Q.E.D.**

Proof of Proposition 2. Assume for contradiction that there exists a separating equilibrium in which the firm offers price p_H to high-preference customers and price p_L to low-preference customers, $p_H \neq p_L$. We first show that $p_L = l$. Note that because all customers will make a no-brainer purchase at $p_L \le l$, the firm has no incentive to offer a low price $p_L < l$. In this sense we have $p_L \ge l$. In addition, to guarantee that low-preference customers are willing to make a purchase, the individual rationality constraint must be satisfied, i.e., $p_L \le l$. This proves that $p_L = l$.

Now consider p_H. Clearly, we must have that $l \le p_H \le h$, this is because customers always (never) purchase at prices below (above) l (h). Moreover, because $p_H \neq p_L$, we have $l < p_H \le h$. Then, a high-preference customer always makes a purchase at price p_H because her expected value exceeds the price:

$$\Omega(p_H)h + \big(1 - \Omega(p_H)\big)l = h \geq p_H.$$

However, given the above strategy, the firm can opportunistically deviate and offer a low-preference customer price p_H instead of p_L. In this case, the firm can make a greater profit from this customer, a contradiction. This proves the proposition. **Q.E.D.**

Proofs of Proposition 3 and 4. See Appendix A. **Q.E.D.**

Proof of Proposition 5. If $c < \bar{c}$, then by Proposition 4, $p_1 = \frac{h+l+\sqrt{(h-l)(h-l-4c)}}{2}$, $p_0 = l$. Since the firm is indifferent between charging the low-preference customers either price, it obtains $(1-\alpha)p_0$ from low-preference customers. In addition, since all high-preference customers purchase after inspection, the firm obtains αp_1 from high-preference customers. Therefore, the total profit is $(1-\alpha)l + \frac{\alpha}{2}\big(h + l + \sqrt{(h-l)(h-l-4c)}\big)$. If $c \geq \bar{c}$, then by Proposition 3, the profit is $\alpha h + (1-\alpha)l$. **Q.E.D.**

Proof of Proposition 6. The result follows from direct profit comparison. **Q.E.D.**

Proof of Proposition 7. The proof follows immediately from the discussions in Section 6.2. First, by Proposition 4, customers obtain zero surplus at either price p_1 or p_0. Second, by Proposition 1, customers obtain positive surplus on average only when the firm offers a regressive price $l + \frac{c}{1-\alpha}$, which arises if and only if $\underline{c} \leq c < \bar{c}$. Therefore, customers are worse-off with personalized pricing with superior knowledge in this region. **Q.E.D.**

Proof of Proposition 8: If $c < \underline{c}$, then by Proposition 7, customer surplus is the same regardless of whether or not the firm possesses superior knowledge. But by Proposition 6, the firm is better off with superior knowledge in this region. Therefore, superior knowledge improves the total social welfare. If $\underline{c} \leq c < \bar{c}$, then by Proposition 4, some low-preference customers may be overcharged with $p_1 > l$ and will reject purchase after inspection. Therefore, there are deadweight losses from inspection under pricing with superior knowledge. However, by Proposition 1, all customers make no-brainer purchases, thus there are no deadweight losses, and superior knowledge reduces the total social welfare in this region. If $c \geq \bar{c}$, then by Proposition 1 and 3, the equilibrium is the same with or without superior knowledge. Therefore, superior knowledge does not affect the total social welfare. **Q.E.D.**

Proof of Proposition 9. We show that the mixed-strategy proposed in Proposition 9 is indeed an equilibrium. First, applying the Bayes' rule, given the signal, the firm's belief about a customer's type is

$$e_H = \Pr[h|\tilde{h}] = \frac{\Pr[\tilde{h}|h]\Pr[h]}{\Pr[\tilde{h}|h]\Pr[h] + \Pr[\tilde{h}|l]\Pr[l]} = \frac{\alpha\beta}{\alpha\beta + (1-\alpha)(1-\beta)},$$

$$e_L = \Pr[h|\tilde{l}] = \frac{\Pr[\tilde{l}|h]\Pr[h]}{\Pr[\tilde{l}|h]\Pr[h] + \Pr[\tilde{l}|l]\Pr[l]} = \frac{\alpha(1-\beta)}{\alpha(1-\beta) + (1-\alpha)\beta}.$$

And given the firm's pricing strategy, a customer's belief is that

$$\Omega(p_1) = \Pr[h|p_1] = \frac{e_H \Pr[\tilde{h}] + \gamma e_L \Pr[\tilde{l}]}{\Pr[\tilde{h}] + \gamma \Pr[\tilde{l}]} = \frac{\alpha\beta + \alpha(1-\beta)\gamma}{\alpha\beta + (1-\alpha)(1-\beta) + \alpha(1-\beta)\gamma + (1-\alpha)\beta\gamma},$$

$$\Omega(p_0) = \Pr[h|p_0] = e_L = \frac{\alpha(1-\beta)}{\alpha(1-\beta) + (1-\alpha)\beta}.$$

Second, consider the customer's inspection and purchasing decisions. It can be shown that, given price p_1, a customer is indifferent between inspecting and making a no-brainer purchase. Mathematically, this presents as

$$\Omega(p_1)h + \left(1 - \Omega(p_1)\right)l - p_1 = \Omega(p_1)(h - p_1) - c = 0.$$

Therefore, the customer is willing to randomize upon receiving the price p_1.

Given price p_0, a customer prefers to not inspect and make a no-brainer purchase. Mathematically, this presents as

$$\Omega(p_0)h + \left(1 - \Omega(p_0)\right)l - p_0 = 0 \geq \Omega(p_0)(h - p_0) - c.$$

Third, consider the firm's pricing decision. Given the customer's behavior, it can be seen that the firm is indifferent between offering a low-signal customer a low price, or tricks her with a high price. Mathematically, we have

$$(1 - \lambda + \lambda e_L)p_1 = p_0.$$

Note that on the left-hand side, $1 - \lambda$ is the probability that the customer will make a no-brainer purchase, and λe_L is the probability that the customer inspects, finds out that her true match value is high, and makes a purchase. Hence, if the firm tricks the customer with p_1, the expected demand from this customer is $1 - \lambda + \lambda e_L$ and the expected profit is $(1 - \lambda + \lambda e_L)p_1$. If the firm offers the customer p_0 instead, the customer will surely make a no-brainer purchase and the firm's profit will be p_0. We can further show that the firm has no incentives to offer prices other than p_1 and p_0, while the details are omitted. **Q.E.D.**

Proof of Proposition 10. In equilibrium, the firm is indifferent between serving the $\tilde{l}$-types with either p_0 or p_1, the profit from this segment is equivalent to the case of no-inspection, in which case the firm obtains $\Pr(\tilde{l})\, p_0$. Since the $\tilde{h}$-types may inspect with the probability of λ, and since in expectation $(1-e_H)$ percent of them may find themselves to be the low-preference consumers and thus reject the price, the firm may only sell to $1-(1-e_H)\lambda$ of the segment, thus the profit is $\Pr(\tilde{h})\, p_1[1-(1-e_H)\lambda]$. Combing these two segments with simple arithmetic, we obtain the expected profit as $\overline{v}+\frac{(2\beta-1)}{\beta}(p_1-\alpha h)$. Note that $p_1=\frac{h+l+\sqrt{(h-l)(h-l-4c)}}{2}<h$, and $\overline{v}$ and p_1 are both independent of β, we have $\frac{\partial\pi}{\partial\beta}=-\frac{\alpha(h-p_1)}{\beta^2}<0$. **Q.E.D.**

Proof of Proposition 11. Given the quality level q, the analysis is analogous to that of the basic model. We can show that in equilibrium, the firm's profit is

$$\pi=\begin{cases} l(1-\alpha)q+\dfrac{\alpha q}{2}\left(h-l+\sqrt{(h-l)\left(h-l-\dfrac{4c}{q}\right)}\right) & \text{if } c<q\alpha(1-\alpha)(h-l), \\ \alpha q h+(1-\alpha)ql & \text{otherwise.} \end{cases}$$

Now consider the case $c=\alpha(1-\alpha)(h-l)-\epsilon$ for some small $\epsilon>0$. If the firm offers $q=1$, then its profit will be

$$\pi_1=l+\alpha^2(h-l)+O(\epsilon).$$

If the firm offers quality $q=\frac{c}{\alpha(1-\alpha)(h-l)}$ instead, its profit will be

$$\pi_2=q(\alpha h+(1-\alpha)l)=l+\alpha(h-l)+O(\epsilon).$$

It follows that $\pi_2-\pi_1=\alpha(1-\alpha)(h-l)+O(\epsilon)$, which is positive when ϵ is small enough. This proves the proposition. **Q.E.D.**