

When Does It Pay to Invest in Pricing Algorithms?

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Abstract

Nowadays, firms frequently use big data and pricing algorithms to offer consumers personalized prices according to their willingness to pay. Advances in information technologies have further facilitated the use of customized pricing, which we consider an important facet of transformative marketing. At the outset, personalized pricing may appear to reduce the asymmetry of information between the firm and consumers, and benefits the firm but hurts consumers. To investigate this view, we consider a novel setting in which consumers must incur search costs to make an informed purchase from the firm. Contrary to conventional wisdom, we find that personalized pricing can sometimes make both the firm and consumers better off, thus leading to a win-win situation. We also show that an imperfect pricing algorithm can outperform a perfect one, thereby explaining why certain retailers like Amazon are adopting imperfect pricing algorithms. On the one hand, a moderately reliable pricing algorithm gives high-preference consumers a chance to be misclassified as low-preference consumers and obtain a low price, thereby encouraging consumer search. On the other hand, a highly reliable pricing algorithm significantly reduces consumers' surplus, which stifles consumer search. As a result, both firm profit and consumer surplus can be non-monotone in the reliability of the algorithm. To the best of our knowledge, this is the first paper that documents the effect of personalized pricing under consumer search.

Keywords: Personalized pricing, pricing algorithm, search, opportunism, holdup.

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1 Introduction

Nowadays, equipped with big-data analytics, firms can easily assess consumers' preferences (i.e., match values) for their products or services from consumers' digital footprints. In the retail industry, Target offers consumers personalized coupons according to their past shopping behavior (Perez, 2019). In the travel and hospitality industries, Orbitz price-discriminates its consumers by tracking their online browsing activities (Mattioli, 2012) while Starwood Hotels & Resorts designs conversational chatbots that can automatically send personalized promotions that induce consumers to accelerate their purchase decisions (Rijmenam, 2017).

Price discrimination with consumer data has been adopted not only by online sellers, but also by brick-and-mortar retailers. For example, cameras and sensors are installed in Amazon's cashierless "Amazon Go" stores to recognize consumers, keep track of their journey in the store, watch what products they are looking at or picking up, and offer them customized coupons (*Coupons in the News*, 2018). In China, stores — particularly luxury brands — are starting to set up cameras at their entrances and use facial recognition to recognize individuals as they enter, and offer consumers personalized prices (Wong, 2018). According to Steve Burd, the CEO of Safeway, "There's going to come a point where our shelf pricing is pretty irrelevant because we can be so personalized in what we offer people" (Choi, 2013).

Advances in information technologies have further facilitated the use of customized pricing, which we consider an important facet of transformative marketing. For instance, facial recognition can facilitate price discrimination in physical settings. This technology can potentially give physical brick-and-mortar stores an additional advantage over online alternatives since the technology is much harder to avoid, which allows potential pricing algorithms to be more reliable. This is unlike many algorithms used in online settings that depend on cookies or user account information, which may be bypassed by clearing cookies or using an alternative/guest account. Facial recognition software may also have additional transformational properties since it can be used to analyze a consumer's facial response when exposed to marketing efforts such as the design of packaging of a product or the way the shelves are organized. Eye-tracking and facial expression analysis can help marketing professionals identify how consumers respond to certain marketing approaches and what they pay the most attention to. This technology can only be reliably implemented in physical locations that is likely not feasible in an online presence since those setting require higher privacy permissions. Pricing algorithms can use facial recognition data to deliver more reliable personalized prices and promotions even as the consumer is shopping. The data can be used to quickly identify a new consumer's preferences who does not have an existing history with the firm and to identify otherwise more idiosyncratic and temporary changes in a known consumer's tastes and preferences while they are shopping. Table 1 summarizes how big-data technologies and pricing algorithms are transforming traditional marketing.

While personalized pricing is becoming more and more prevalent among firms, consumers are often uneasy about it. As stated in the 2015 White House report on big data, there is a general concern that personalized pricing "transfers value from consumers to shareholders, which gen-

Table 1: Personalized Pricing: A Facet of Transformative Marketing

	Transformative Marketing	Traditional Marketing
Data	The firm has individual level consumer data (e.g., through cookies and facial recognition)	The firm has store level consumer data (e.g., through IRI, AC Nielsen)
Firm's Information	The firm can obtain information about individual consumers' preferences	The firm does not know the preferences of individual consumers
Pricing	The firm is able to offer personalized prices to different consumers (i.e., first-degree price discrimination)	The firm offers all consumers a uniform price (or adopt second- or third-degree price discrimination)
Consumers' Information	A consumer does not observe the prices charged to other consumers	A consumer observes the prices charged to other consumers
Consumer Reactions	The firm observes individual consumers' response to customized prices	The firm observes consumers' response to the price offerings

erally leads to an increase in inequality and can therefore be inefficient," and moreover, "This is particularly true in settings where there is no competition, and few consumers would exit the market, even if a firm raised prices dramatically" (*Council of Economic Advisers*, 2015). Several surveys indicate that consumers' awareness about and resentment towards price discrimination has also grown steadily (Turow et al., 2005; Edwards, 2006). While personalized pricing allows a firm to price discriminate amongst its consumers, consumers may refrain from visiting the firm to avoid being over-exploited by the algorithm, which can backfire on firm profit. Collectively, it is unclear how personalized pricing affects a firm's profitability.

In this paper, we consider a monopolistic firm that identifies consumers with an imperfect profiling algorithm. We assume that consumers must incur a search cost to be able to make an informed purchase from the firm. For example, consumers often need to incur significant costs both online and offline (e.g., transportation costs and visiting costs) to arrive at the store and evaluate the products. Contrary to conventional wisdom that personalized pricing benefits a firm and hurts consumers, we show that when there is a positive search cost, personalized pricing can make both the firm and consumers better off or worse off, thereby leading to a win-win or a lose-lose situation. We also show that in the presence of a search cost, an imperfect pricing algorithm can outperform a perfect one, thereby explaining why certain retailers such as Amazon are adopting imperfect pricing algorithms (Solon, 2011). To the best of our knowledge, this is the first paper that studies the effect of personalized pricing in the presence of consumer search. This research will help firms better design their personalized pricing practice and guide public policymakers to better regulate the industry.

Preview of Findings

Based on our model results, we make a number of observations. First, we demonstrate that consumers' search decisions can be affected by the reliability of the firm's pricing algorithm. On the one hand, a moderately reliable algorithm encourages the firm to price discriminate its consumers and offer them personalized prices, while leaving high-valuation consumers a reasonable chance to be misclassified as low-valuation consumers and obtain a low price. This chance of underpricing induces consumer search. On the other hand, a highly reliable algorithm allows the firm to fully exploit consumer surplus through first-degree price discrimination while leaving consumers little surplus, thereby stifling consumer search. Overall, the profiling algorithm can either induce or deter consumer search, and consumers are more likely to search when the reliability of the algorithm is within a certain range.

Second, while common wisdom holds that a more reliable pricing algorithm will (weakly) benefit the firm, we find that both the firm's profit and consumer surplus are not monotone in the reliability of the algorithm. To understand the above result, note that the effect of an increase in the algorithm's reliability has both an upside and a downside. On the upside, it allows the firm to better price discriminate its consumers. On the downside, it may stifle consumer search. As a result, the firm's profit is maximized when the reliability of its algorithm is relatively high but not too high. At such reliability, the firm takes advantage of the price discrimination effect while at the same time inducing consumer search. For consumers, under certain circumstances, their surplus is maximized when the reliability of the algorithm is low but not too low, which leaves high-preference consumers a good chance to be misclassified while still keeping the firm interested in price discriminating its consumers.

Third, we endogenize the firm's decision in developing the pricing algorithm, where the firm incurs a fixed cost to develop the algorithm. We find that the firm develops the pricing algorithm only when the algorithm is moderately reliable. In essence, an unreliable algorithm does not help the firm, while a highly reliable algorithm stifles consumer search.

Background

Our paper is related to the literature on consumer addressability, which refers to the identification of specific consumers. As mentioned by Acquisti and Varian (2005), consumer addressability is essentially a form of first-degree price discrimination, using identity as a signal of willingness-to-pay. While it may be intuitive that consumer addressability helps a monopolistic firm obtain higher profit through price discrimination, researchers find the opposite in competitive markets. Thisse and Vives (1988) study the competition between firms that are located at different spaces. They consider a setup where the firms have the ability to address all consumers in the market and charge them customized prices, and show that consumer addressability intensifies market competition, leaving both firms worse off. Bester and Petrakis (1996) and Choudhary et al. (2005) have examined one-to-one promotions and other forms of personalized pricing, and they find

that personalized pricing generally leads to a form of prisoner's dilemma: all firms are worse off compared to the case where they cannot offer personalized prices. Xu and Dukes (2019) study a scenario in which firms have superior information about consumers' willingness to pay, and show that consumer suspicion can affect the firms' pricing and product line design decisions. Li and Xu (2022) assume the firm knows a consumer's valuation perfectly while the consumer must incur an inspection cost to find her valuation, and discuss the firm's optimal pricing decision. In contrast, we endogenize both the firm's decision to invest in a pricing algorithm as well as the reliability of the firm's pricing algorithm, key decisions not considered by Li and Xu (2022).

While all the above papers assume that firms always address consumers perfectly, some marketing and operations management studies relax this assumption and consider imperfect targetability. Chen et al. (2001) consider a competitive market where competing firms are endowed with imperfect individual targeting technologies which allow the firms to classify consumers into either price-sensitive switchers or price-insensitive loyal consumers. Due to imperfect targetability, each firm mistakenly perceives some switchers as loyal customers and charges them all a higher price. They find that the competitive advantage that a firm gains from its improved targetability need not be detrimental to its rival firm. This result arises because, as a firm becomes better at distinguishing its loyal consumers from switchers, it gains the confidence to charge a higher price to its perceived loyal customers, which alleviates market competition. Chen and Iyer (2002), in contrast, assume that firms can perfectly identify consumers that are reached, but that they have imperfect reach. They show that it may not be in the best interest of the firms to reach all consumers in the market. Koh et al. (2017) investigate an imperfect profiling technology which fails to profile a consumer with a certain probability. Valletti and Wu (2020) consider a case in which a firm profiles consumers using an imperfect technology, and that the firm's technology becomes more accurate when more consumers participate in profiling. They show that consumer surplus and social welfare are non-monotone in consumers' ability to conceal their identities.

It is well established in the literature that product search can be costly to consumers. For example, Hauser and Wernerfelt (1990) argue that, because of the lack of knowledge and variation among consumption occasions, a consumer must incur an evaluation cost when considering alternative products. Mehta et al. (2003) provide empirical support that consumers incur significant search costs prior to price discovery. As the search cost is incurred prior to price discovery, the firm does not take search cost into account when choosing its price (Lal and Matutes, 1994; Villas-Boas, 2009; Anderson and Renault, 2006).

Our research contributes to the above literature streams by considering the effect of personalized pricing on consumer search (see Table 2 for a summary). We show that personalized pricing can either aggravate or alleviate the holdup issue due to consumer search costs. We show that when consumers incur a positive search cost to make a purchase, an imperfect pricing algorithm dominates a perfect algorithm and is better than not implementing price discrimination, both from firm and consumer standpoints (in Appendix A we show the importance of studying both consumer search and personalized pricing in the framework). Therefore, firms must take consumer

search into consideration when deciding whether or not to implement personalized pricing.

Table 2: Comparing our paper against the literature

	Personalized Pricing	Consumer Search	Endogenous Algorithm	Imperfect Algorithm
Thisse and Vives (1988)	Yes	No	Yes	No
Bester and Petrakis (1996)	Yes	No	No	No
Choudhary et al. (2005)	Yes	No	No	No
Xu and Dukes (2019)	Yes	No	No	No
Li and Xu (2022)	Yes	No	No	No
Hauser and Wernerfelt (1990)	No	Yes	No	No
Mehta et al. (2003)	No	Yes	No	No
Kim et al. (2010)	No	Yes	No	No
This paper	Yes	Yes	Yes	Yes

Note: Li and Xu (2022) assume that consumers must incur an inspection cost to find out their true preferences. Here, the inspection cost differs from search cost because consumers incur the search cost before observing the price, while in Li and Xu (2022), consumers first observe the price and then decide whether to inspect.

2 The Model

Consider a monopolist firm selling a product to consumers. The marginal production cost for the product is constant and normalized to zero without loss of generality. There is a unit mass of consumers with unit demand for the product. Consumers are forward looking and their valuations for the product are i.i.d. with the following two-point distribution,

$$v_i = \begin{cases} H & \text{with probability } \alpha, \\ L & \text{with probability } 1 - \alpha, \end{cases}$$

where $H > L > 0$. Here v_i is consumer i 's willingness to pay (or match value) for the product which is unknown to the consumer prior to search. For simplicity, we refer to consumers with high valuations as the high-type consumers and others as the low-type consumers. The prior distribution of v is common knowledge for the firm and the consumers. Note that here we implicitly assume that a consumer's valuation is always above the marginal production cost, and therefore it is socially efficient for transactions to take place.

Consumer Search. Before visiting the store (either online and offline), consumers are poorly informed about the price and attributes of the product, and do not know whether the product matches their preferences or not (Liu et al., 2019; Chen, 2021). Following the literature (Anderson and Renault, 2006; Villas-Boas, 2009; Guo and Meng, 2014), we assume that consumers must incur a search cost (or travelling cost), s , to be able to make a purchase. The cost is sunk when consumers are already at the firm's location.

In Web Appendix ?? we consider the case in which consumers can have heterogeneous search costs. We show that all the main results are not qualitatively altered under heterogeneous search costs.

Pricing algorithm. Consider a firm’s decision to invest in the development of a pricing algorithm which allows it to classify and profile individual consumers based on their willingness to pay for the product. If the firm chooses to develop the algorithm, it has a better understanding of consumers’ valuations before they visit the store (online or offline) because the firm knows not only consumers’ preferences (e.g., through analyzing their purchase data, IP addresses, click-stream information, and facial recognition), but also the attributes of the product. The consumers, in contrast, are less informed about the attributes of the product before visiting the firm, and therefore do not know their match value for the product.

The firm’s pricing algorithm may be imperfect. As discussed by Chen et al. (2001), because of imperfect customer information or processing technology, the firm does not always classify individual consumers correctly. For example, the firm may misclassify a high-type (low-type) consumer as a low-type (high-type) consumer and charge her a low (high) price. An imperfect pricing algorithm reflects the marketplace reality. As suggested by *Council of Economic Advisers* (2015), when big data suggests that a consumer is 75 percent pregnant, there is a 25 percent risk of identifying a “false positive” consumer.

To model such imperfection, we consider the following specification for the firm’s pricing algorithm. For each consumer i , the algorithm provides the firm with a signal θ_i about that consumer’s valuation, which is either “high” (h) or “low” (l), i.e., $\theta_i \in \{h, l\}$. Following Chen et al. (2001) and Iyer et al. (2007), we operationalize the reliability (or accuracy) of the profiling algorithm by a measure $\gamma \in [0, 1]$ and assume that

$$\Pr(h|H) = \Pr(l|L) = \frac{1}{2} + \frac{\gamma}{2}, \quad \Pr(l|H) = \Pr(h|L) = \frac{1}{2} - \frac{\gamma}{2}.$$

The above specification satisfies the following properties: (i) When $\gamma \rightarrow 1$, the profiling algorithm is perfect, i.e., the signal reveals the true valuation with certainty: $\Pr(H|h) = \Pr(L|l) = 1$, (ii) When $\gamma \rightarrow 0$, the signal offers no additional information beyond the prior: $\Pr(H|h) = \Pr(H) = \alpha$ and $\Pr(L|l) = \Pr(L) = 1 - \alpha$, (iii) The likelihood of misclassification, $\Pr(l|H)$ and $\Pr(h|L)$, decreases in γ .

It is worth mentioning that our specification implies that $\Pr(h|H) = \Pr(l|L)$, i.e., the algorithm performs equally well in profiling high and low-type consumers. In Web Appendix ??, we allow the algorithm to have asymmetric performances on the two consumer types.

Given the algorithm described above, when a consumer arrives at the firm’s store (online or offline), the firm obtains a signal and updates its belief about the consumer’s valuation following the Bayes’ rule, that is,

$$\Pr(H|h) = \frac{\alpha + \alpha\gamma}{1 + 2\alpha\gamma - \gamma}, \quad \Pr(H|l) = \frac{\alpha - \alpha\gamma}{1 - 2\alpha\gamma + \gamma}.$$

The firm then optimizes its pricing strategy accordingly.

Algorithm development. We allow the firm to decide whether or not to develop a pricing algorithm with reliability γ . Here γ represents the state-of-the-art technology limit. Nonetheless, in Web Appendix ??, we allow the firm to also choose the reliability of its algorithm and show that our findings still hold. Let $F \geq 0$ denote the fixed cost of developing the algorithm, e.g., investing in developing a database that is capable of consumer profiling and hiring scientists to develop the algorithm (see Liu and Zhang (2006) and Matsumura and Matsushima (2015)). This is consistent with the observation that “firms are making massive investments into building information infrastructures that allow them to collect, store, and analyze consumer data” (Acquisti and Varian, 2005). If the firm does not pay this cost, it cannot price discriminate its consumers.

Timing of the Game. The game unfolds in four stages. In Stage 0, the firm decides whether or not to develop the pricing algorithm. If it develops the algorithm, it pays the development cost F . Otherwise, the firm saves the cost but cannot assess any information about consumer valuation beyond its prior. In Stage 1, consumers decide whether or not to visit the firm’s store (online or offline). If a consumer visits the firm’s location, she must incur a search cost $s > 0$, which is necessary for purchase. In Stage 2, the firm offers prices to consumer arrivals. There are two cases depending on whether or not the firm has developed the algorithm. If the firm did not develop the algorithm, it offers a uniform price p to all consumer arrivals. Otherwise, the algorithm generates a signal $\theta_i \in \{h, l\}$ for each consumer arrival (either online or a physical store) i about her willingness to pay for the product, and the firm offers a price p_{θ_i} to that consumer, which is optimal given the algorithm γ and signal θ_i . In Stage 3, each such consumer makes her purchase decision.

The price setting process represents the firm offering consumers customized coupons when they arrive at its online or physical stores. This is similar to a firm choosing the price prior to consumers’ search decisions (i.e., the firm obtains the signal and chooses a price for each consumer in its database even before the consumer visits the firm’s online or physical stores). Since consumers observe the price after searching, this difference in model timing does not affect the equilibrium outcome. In other words, despite some differences in the solution concepts, the two model setups essentially lead to the same equilibrium outcome.

3 Exogenous Algorithm

In this section, we analyze the case in which the firm’s algorithm is exogenous. We then endogenize the firm’s algorithm development decision in Section 4.

Here, we assume that consumers know the firms’ algorithm reliability before they search. Similar assumptions are commonly made in the search literature (Guo and Meng, 2014; Villas-Boas, 2009), and we relax this assumption in Section 4. We characterize the equilibrium through the process of backward induction where we start with the last decision node in the game — the firm’s pricing strategy for the product when consumers are already at the firm’s online or physical

location.

3.1 Optimal Pricing Strategies

Let us consider a consumer's purchase decision when she already incurs the search cost and arrives at the firm's location. If the consumer does not make a purchase, her payoff will be $-s$. If she makes a purchase, her payoff will be $-s + (v_i - p_i)$. Thus, the consumer makes a purchase if and only if the price of the good does not exceed her valuation, i.e., $p_i \leq v_i$.

Now consider the firm's optimal pricing strategy given its pricing algorithm. As described above, for each consumer arrival i , the algorithm generates a signal θ_i indicating the consumer's valuation v_i for its product. The firm's pricing strategy thus depends on this signal θ_i . Let $p_h(p_l)$ be the firm's optimal price upon observing signal $\theta_i = h(\theta_i = l)$.

The firm's optimal pricing strategy involves the following two cases, depending on the segment size of the high-type consumers.

Case 1. $\alpha H > L$.

Consider first the case where the segment size of the high-type consumers is large enough, i.e., $\alpha H > L$.

Because consumers do not know the product's price before arriving at the firm, the price has no effect on consumers' purchase decision. Therefore, the firm does not take consumers' search cost into account when making its pricing decisions. When consumers are already at the store, as described above, the search cost is already sunk and a consumer is willing to buy as long as $v_i \geq p_i$. Without the algorithm, the firm charges a single price $p = H$ to all consumer arrivals, serving only high-type consumer arrivals for a high margin. It makes an expected profit of αH from each consumer. With the algorithm, depending on the reliability of the algorithm, there are two cases:

- When

$$\gamma \leq \gamma_1 = (\alpha H - L) / (\alpha H + L - 2\alpha L), \quad (1)$$

the firm charges a single price $p_h = p_l = H$ to all consumers.

- Otherwise, the firm charges an h -signal consumer a price $p_h = H$ and an l -signal consumer a price $p_l = L$.

The intuition for the above pricing strategy is as follows. When the algorithm is unreliable (i.e., $\gamma < \gamma_1$), a large portion of the l -signal consumers are actually high-type consumers who are misclassified by the algorithm, i.e., $\Pr(H|l)$ is high. Given the large volume of high-type consumers in the l -signal, it is optimal for the firm to ignore the signal and offer the l -signal consumers a high price and only serve the high-type consumers. As the algorithm becomes more reliable, the likelihood of "false negative" decreases, i.e., $\partial \Pr(H|l) / \partial \gamma < 0$. That is, an l -signal consumer is

likely to be indeed a low-type consumer, and it becomes optimal for the firm to use the signal and offer l -signal consumer a low price.

Case 2. $\alpha H \leq L$.

Second, consider the case in which the segment size of high-type consumers is small. Depending on the reliability of the algorithm, there are two cases:

- When

$$\gamma \leq \gamma_2 = (L - \alpha H) / (\alpha H + L - 2\alpha L), \quad (2)$$

the firm charges a single price $p_h = p_l = L$ to all consumers.

- Otherwise, the firm charges an h -signal consumer a price $p_h = H$ and an l -signal consumer a price $p_l = L$.

As described above, when the algorithm is sufficiently unreliable, the likelihood of “false positive”, i.e., $\Pr(L|h)$, is so high that the firm ignores the signal and offers a low price $p_h = L$ to an h -signal consumer. When the algorithm is reliable enough, the likelihood of false positive is low and the firm offers a high price to an h -signal consumer, i.e., $p_h = H$.

3.2 Consumers' Search Decision

In the above analysis, we have derived the firm's optimal pricing strategies. Now, consider consumers' search decision. In equilibrium, given γ , forward-looking consumers correctly anticipate the price that the firm is going to charge and take it into account in their search decisions. Because all consumers share the same prior belief and search cost, they adopt the same search strategy. To break ties, we assume that when a consumer is indifferent about searching, she always searches. This tie-breaking rule works to the advantage of the firm and is consistent with Pareto dominance.

Again, our analysis involves the following two cases.

Case 1. $\alpha H > L$.

Before characterizing the consumers' search strategy, it is useful to consider two benchmark cases: (i) The firm does not use profiling algorithms (or the firm's algorithm is completely unreliable), and (ii) the firm's algorithm is perfect.

Without a pricing algorithm, the firm offers a single price $p = H$ to all consumers. Such a pricing strategy expropriates the entire consumer surplus. Anticipating this, consumers are reluctant to search and there will not be any sales.

When $\gamma = 1$, the algorithm is perfect in the sense that $\Pr(H|h) = \Pr(L|l) = 1$. As discussed above, the firm always offers price $p_h = H$ to high-type consumer arrivals and price $p_l = L$ to low-type consumer arrivals. Now consider the consumers' search decision. If a consumer searches, her expected surplus will be $CS = -s + \alpha(H - p_h) + (1 - \alpha)(L - p_l) = -s$; if she does not search,

her surplus will be $CS = 0$. Because consumer surplus is higher in the latter case, the consumer prefers to not search and the firm cannot make a sale.

From the above analysis we show that consumers exit the market when the algorithm is completely unreliable or perfectly reliable. Is it possible that a moderately reliable algorithm induces consumers to search? Lemma 1 specifies the equilibrium search behavior given any algorithm $\gamma \in [0, 1]$. All proofs are in Appendix B.

Lemma 1 *Suppose that $\alpha H > L$. Consumers will search if and only if the algorithm is moderately reliable, i.e.,*

$$\gamma_1 \leq \gamma \leq \gamma_3 = 1 - 2s/(\alpha H - \alpha L). \quad (3)$$

Lemma 1 suggests that given a positive search cost, consumers are willing to search if the algorithm is moderately reliable. Consider the following numerical example: $H = 3$, $L = 1$, $\alpha = 0.5$ and $s = 0.1$. That is, consumers are willing to search as long as $0.33 \leq \gamma \leq 0.8$, but are otherwise reluctant to search. But why is this so?

Consider the case in which the algorithm is sufficiently unreliable, i.e., $\gamma < \gamma_1$. As mentioned above, the firm completely ignores the signal the algorithm generates, and offers all consumers the same price, $p = H$. However, given this pricing strategy, a consumer does not derive any benefit from visiting the firm's store (online or bricks and mortar). Alternatively, consider the case of a highly reliable algorithm, i.e., $\gamma > \gamma_3$. As the algorithm is so reliable, the firm leverages the technology and bases its price offerings on the signal generated by the algorithm. The algorithm substantially reduces the asymmetry of information between the firm and the consumers, as well as the consumers' surplus. When the algorithm is reliable enough, consumers' expected benefit from search becomes so low that it falls short of their search cost. In either case, consumers do not search or make purchases, but for different reasons.

Finally, consider the case of a moderately reliable algorithm, i.e., $\gamma_1 < \gamma < \gamma_3$. As the reliability is not too low, the firm is willing to leverage the algorithm and tailor its price offerings based on the signal that it receives from the algorithm, i.e., it offers an h -signal consumer a price $p_h = H$ and an l -signal consumer a price $p_l = L$. Interestingly, since the algorithm is imperfect, a high-type consumer might be misclassified as an l -signal consumer and subsequently enjoy a deal of $p_l = L$. When $\gamma < \gamma_3$, the likelihood of misclassification and underpricing is significant, leaving consumers a reasonable surplus which compensates for their search cost. Mathematically, this result presents as $\Pr(H \wedge l)(H - L) \geq s$, where $\Pr(H \wedge l) = \alpha \Pr(l|H)$ is the probability that a consumer has a high valuation but is misclassified as an l -signal consumer. Thus, the moderately reliable algorithm allows market efficiency, thereby benefiting both the firm and consumers.

Case 2. $\alpha H \leq L$.

Now, consider the case in which the segment size of high-type consumers is low. Without an algorithm, the firm offers price $p = L$ to all consumer arrivals. It follows immediately that con-

sumers are willing to search if and only if the search cost does not exceed their expected benefit, i.e., $s \leq \alpha(H - L)$.

What happens when the firm is endowed with an algorithm of reliability, γ ? The equilibrium consumer search behavior is described in the following lemma.

Lemma 2 *Suppose that $\alpha H \leq L$. Given the reliability of the algorithm, consumers' equilibrium search behavior is described below, where γ_2 and γ_3 are defined in (2) and (3).*

- (1) *If $s > \alpha(H - L)$, consumers never search.*
- (2) *If $\alpha(1 - \gamma_2)(H - L)/2 < s \leq \alpha(H - L)$, consumers will search if and only if $\gamma < \gamma_2$.*
- (3) *If $s \leq \alpha(1 - \gamma_2)(H - L)/2$, consumers will search if and only if $\gamma \leq \gamma_3$.*

Part (1) of Lemma 2 says that if search cost is prohibitive, consumers do not visit the firm's online or physical store regardless of the reliability of its algorithm. This result arises as the potential gain from shopping always falls short of the high search cost.

Part (2) suggests that, when $\alpha(1 - \gamma_2)(H - L)/2 < s \leq \alpha(H - L)$, consumers are willing to search if and only if the algorithm is sufficiently unreliable, i.e., $\gamma \leq \gamma_2$. Note that when $\gamma \leq \gamma_2$, the algorithm is so unreliable that the firm ignores the signal generated by the algorithm and charges a single price $p = L$ to all consumers. As such, a high-type consumer always enjoys a low price upon visiting the firm. The expected gain covers the search cost and consumers are willing to visit the firm. If $\gamma > \gamma_2$, however, the algorithm becomes relatively reliable and the firm starts to almost perfectly price discriminate amongst its consumers, i.e., it offers h -signal consumers a high price, $p_h = H$, and l -signal consumers a low price, $p_l = L$. As the firm starts to price discriminate almost perfectly, consumers' benefit from search falls short of their search cost and they exit the market.

It is worth noting that a highly reliable algorithm stifles consumer search and resulting in no sales: When consumers are already at the firm's location (online or offline), the firm only cares about capturing consumers' surplus through price discrimination. The firms' ex post opportunism, however, is suboptimal from an ex ante perspective.

Part (3) speaks to the case in which the search cost is low enough, i.e., $s \leq \alpha(1 - \gamma_2)(H - L)/2$. As discussed above, if $\gamma \leq \gamma_2$, the algorithm is so unreliable that the firm ignores the algorithm and does not price discriminate its consumers. As such, consumers benefit from search and are therefore willing to search. If $\gamma > \gamma_2$, the algorithm is more reliable and the firm price discriminates consumers with the algorithm. In this case, a consumer derives a positive surplus if and only if she is a high-type consumer but is misclassified as an l -signal consumer. That is, the expected surplus a consumer obtains after arriving at the firm's location is $\Pr(l \wedge H)(H - L) = \alpha \Pr(l|H)(H - L)$. When $\gamma \leq \gamma_3$, $\Pr(l|H)$, the likelihood of misclassification and hence the expected consumer surplus is high enough, and therefore consumers are willing to search. When $\gamma > \gamma_3$, however, the highly reliable algorithm reduces the likelihood of misclassification, leaving

consumers with little benefit. In anticipation of this, consumers are reluctant to visit the firm (online and offline) and thus exit the market. Mathematically, this arises as $\partial \Pr(l|H)/\partial \gamma = -\frac{1}{2} < 0$.

3.3 Firm Profit and Consumer Surplus

In the analysis above, we have characterized the effect of the algorithm on the firm's pricing strategy and consumers' search decisions. In this section, we further discuss the implications of the algorithm for firm profit and consumer surplus. Again, we consider the following two cases.

Case 1. $\alpha H > L$.

Consider first the case in which the segment size of high-type consumers is large enough. Following the firm's pricing strategy and Lemma 1, we present the following proposition.

Proposition 1 *Suppose that $\alpha H > L$ and the reliability of the algorithm is γ . The equilibrium firm profit and consumer surplus are presented below.*

(i) *If $s > \alpha(1 - \gamma_1)(H - L)/2$, firm profit and consumer surplus are always zero.*

(ii) *If $s \leq \alpha(1 - \gamma_1)(H - L)/2$, firm profit is given by*

$$\pi = \begin{cases} 0 & \text{when } \gamma < \gamma_1 \text{ or } \gamma > \gamma_3, \\ \frac{\alpha(1+\gamma)H}{2} + \frac{(1+\gamma-2\alpha\gamma)L}{2} & \text{when } \gamma_1 \leq \gamma \leq \gamma_3, \end{cases}$$

and consumer surplus is given by

$$CS = \begin{cases} 0 & \text{when } \gamma < \gamma_1 \text{ or } \gamma > \gamma_3, \\ \frac{\alpha(1-\gamma)(H-L)}{2} - s & \text{when } \gamma_1 \leq \gamma \leq \gamma_3. \end{cases}$$

Firm profit (consumer surplus) is maximized at $\gamma = \gamma_3$ ($\gamma = \gamma_1$).

Could a more reliable pricing algorithm hurt the firm? In the absence of search costs, the answer would be an ambiguous "no": a more reliable algorithm reduces the asymmetry of information between the firm and the consumers, which only benefits the firm. But this is not necessarily so under positive search costs: as we noted, a reliable algorithm reduces consumers' surplus and stifles search, which destructs value.

As shown in Proposition 1, when consumers' search costs are not too high, i.e., $s \leq \alpha(1 - \gamma_1)(H - L)/2$, both firm profit and consumer surplus are non-monotone in the reliability of the algorithm. Moreover, firm profit is maximized at $\gamma = \gamma_3$, whereas consumer surplus reaches its maximum at $\gamma = \gamma_1$, both of which are interior solutions. This result is in contrast with common wisdom that a pricing algorithm always benefits a firm at the expense of consumers.

To illustrate the above findings, we plot firm profit in Figure 1 ($H = 3, L = 1, \alpha = 0.5$, and $s = 0.1$). As we can see from Figure 1, the firm makes zero profits when its algorithm is either

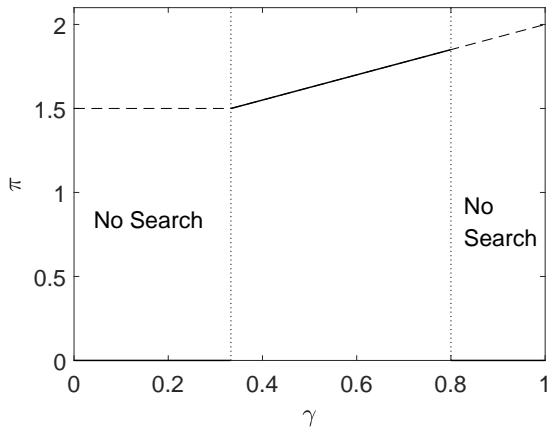


Figure 1: Equilibrium firm profit
($H = 3, L = 1, \alpha = 0.5, s = 0.1$)

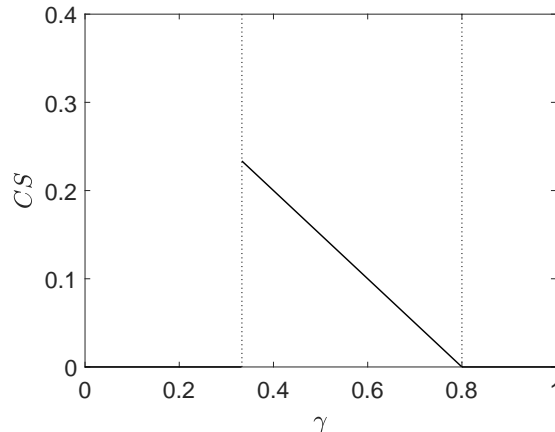


Figure 2: Equilibrium consumer surplus
($H = 3, L = 1, \alpha = 0.5, s = 0.1$)

unreliable enough ($\gamma < \gamma_1 \approx 0.33$) or reliable enough ($\gamma > \gamma_3 = 0.8$). In contrast, the firm's profit is positive and increases with γ when the reliability of its algorithm is moderate ($\gamma_1 \leq \gamma \leq \gamma_3$), and there is a discontinuous increase (decrease) in firm profit at $\gamma = \gamma_1$ ($\gamma = \gamma_3$).

When consumers always search (or equivalently, when $s = 0$), the firm is (weakly) better off with a higher γ . This is because a more reliable algorithm reduces the asymmetry of information between the firm and the consumers, which allows the firm to expropriate consumer surplus through price discrimination. This result is illustrated with the dashed line in Figure 1, where firm profit (weakly) increases with γ . This result, however, does not necessarily hold when consumers incur positive search costs to make an informed purchase. As we can see from the solid line in Figure 1, when the algorithm is sufficiently reliable or sufficiently unreliable, consumers expect a low surplus from visiting a firm's store (online or offline) and are thus reluctant to search. As a result, consumers exit the market.

Similarly, we plot the equilibrium consumer surplus in Figure 2. As indicated by Figure 2, consumer surplus peaks at $\gamma = \gamma_1$. When the reliability falls below γ_1 , the firm forgoes the algorithm and charges a high price, $p = H$, to all consumers, which stifles consumer search at the outset. Consider now the case in which $\gamma \geq \gamma_1$. As γ increases, the algorithm becomes more reliable, reducing the asymmetry of information between the firm and consumers as well as consumers' benefit from search. As a result, consumer surplus declines. Finally, at $\gamma = \gamma_3$, the expected benefit falls short of the search cost and consumers cease to visit the firm's store (online or offline).

The above results suggest that, under positive search costs, the algorithm partly aligns the interests of the firm and consumers: they both benefit from an algorithm that is moderately reliable ($\gamma_1 \leq \gamma \leq \gamma_3$). Within this regime, the algorithm helps alleviate the holdup problem caused by consumer search, thereby improving social efficiency and leading to a win-win situation. The result — that both the firm and consumers can benefit from the algorithm — is in stark contrast to the no search cost case in which the interests of the firm and consumers are perfectly misaligned

(in the sense that the algorithm always benefits the firm at the expense of consumers.)

It is worth noting that there exist other ways for the firm to mitigate the holdup problem, which are excluded from our model setup. For instance, it is well-established that price advertising can help a firm commit to a low price, thereby incentivizing consumer search (Lal and Matutes, 1994). Nonetheless, not all firms price advertise, and not all consumers pay attention to a firm's price advertising. In addition, when a firm price advertises, it commits to charging a *uniform* price to all consumers, which restricts the firm's pricing ability. Indeed, we can show that under various situations, using a pricing algorithm can be more profitable than price advertising.

Case 2. $\alpha H \leq L$.

Next, consider the case in which the segment size of high-type consumers is small. The following proposition follows from the firm's pricing strategy and Lemma 2. When there are multiple equilibria, we use Pareto dominance to select the dominant equilibrium.

Proposition 2 *Suppose that $\alpha H \leq L$ and the reliability of the algorithm is γ . The equilibrium firm profit and consumer surplus are presented below.*

(i) *If $s > \alpha(H - L)$, firm profit and consumer surplus are always zero.*

(ii) *If $\alpha(1 - \gamma_2)(H - L)/2 < s \leq \alpha(H - L)$, firm profit is given by*

$$\pi = \begin{cases} L & \text{when } \gamma \leq \gamma_2, \\ 0 & \text{otherwise,} \end{cases}$$

and consumer surplus is given by

$$CS = \begin{cases} \alpha(H - L) - s & \text{when } \gamma \leq \gamma_2, \\ 0 & \text{otherwise.} \end{cases}$$

Both firm profit and consumer surplus are maximized when $\gamma \leq \gamma_2$.

(iii) *If $s \leq \alpha(1 - \gamma_2)(H - L)/2$, firm profit is given by*

$$\pi = \begin{cases} L & \text{when } \gamma \leq \gamma_2, \\ \frac{\alpha(1+\gamma)H}{2} + \frac{(1+\gamma-2\alpha\gamma)L}{2} & \text{when } \gamma_2 < \gamma \leq \gamma_3, \\ 0 & \text{otherwise,} \end{cases}$$

and consumer surplus is given by

$$CS = \begin{cases} \alpha(H - L) - s & \text{when } \gamma \leq \gamma_2, \\ \frac{\alpha(1-\gamma)(H-L)}{2} - s & \text{when } \gamma_2 < \gamma \leq \gamma_3, \\ 0 & \text{otherwise.} \end{cases}$$

Firm profit (consumer surplus) is maximized at $\gamma = \gamma_3$ ($\gamma \leq \gamma_1$).

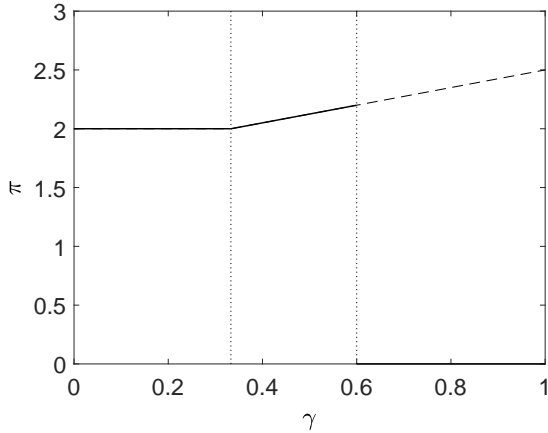


Figure 3: Equilibrium firm profit
($H = 3, L = 2, \alpha = 0.5, s = 0.1$)

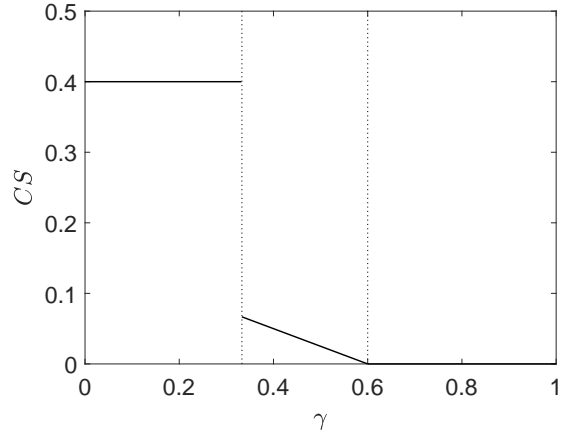


Figure 4: Equilibrium consumer surplus
($H = 3, L = 2, \alpha = 0.5, s = 0.1$)

Again, Proposition 2 shows that, under positive search costs, firm profit is non-monotone with the reliability of the algorithm. An increase in γ not only improves the firm's pricing power, but also makes search less profitable. While the former effect works to the benefit of the firm, the latter effect works to its detriment and may outweigh the former. As for consumers, their surplus always decreases with the reliability of the algorithm, as a more reliable algorithm reduces their benefit from search. Interestingly, under positive search costs, the interests of the firm and the consumers are partly aligned: neither the firm nor the consumers prefer a highly reliable algorithm which inefficiently damages the market.

Propositions 1 and 2 suggest that a highly reliable algorithm stifles consumer search with consumers exiting the market. Both the firm and consumers would be better off if the firm commits to forfeiting the algorithm. Nevertheless, such a commitment would be unconvincing: Even though it is ex-ante rational for the firm to forfeit the algorithm and induce consumers to search, it is not rational ex-post. When consumers are already at the firm's location, the firm always has an incentive to implement the (most reliable) algorithm and expropriate consumer surplus as much as possible. Such ex-post opportunism by the firm stops consumer search at the outset, which backfires on the firm's profit.

The above discussion implies that, interestingly, a regulation that bans information collection or consumer profiling can help the firm credibly commit to not using the algorithm, which can benefit both the firm and consumers.

4 Endogenous Algorithm Development Decision

In this section, we consider the full model in which the firm decides whether or not to invest in a pricing algorithm. We assume that the firm's investment decision is not observed by consumers. Here, the reliability of the pricing algorithm is set at γ ; that is, if the firm invests in the algorithm, the reliability of its algorithm will be γ . If it does not invest, then it does not acquire any infor-

mation about consumer preferences beyond the prior. We extend our model to the case where the firm can choose any reliability level in the Web Appendix.

4.1 A Perfect Information Benchmark

Before analyzing the model, it is useful to consider a perfect information benchmark in which the firm's development decision is perfectly observed by consumers. That is, consumers observe whether or not the firm develops the pricing algorithm.

Case 1. $\alpha H > L$.

The following lemma summarizes the firm's optimal development strategy. In case of multiple equilibria, we use Pareto dominance to select the dominant equilibrium.

Lemma 3 *Suppose that $\alpha H > L$ and that consumers perfectly observe the firm's development decision. The firm develops the algorithm if and only if $s \leq \alpha(1 - \gamma_1)(H - L)/2$, $\gamma_1 \leq \gamma \leq \gamma_3$ and $F \leq V_1^p = \alpha(1 + \gamma)H/2 + (1 + \gamma - 2\alpha\gamma)L/2$.*

Lemma 3 suggests that the firm is willing to develop the algorithm when the algorithm is moderately reliable and the development cost is low. On the one hand, an unreliable algorithm does not help the firm fine-tune its pricing decision. On the other hand, a highly reliable algorithm reduces consumers' surplus and stifles search.

Case 2. $\alpha H \leq L$. Lemma 4 characterizes the equilibrium outcome.

Lemma 4 *Suppose that $\alpha H \leq L$ and that consumers perfectly observe the firm's development decision. The firm develops the algorithm if and only if $s \leq \alpha(1 - \gamma_2)(H - L)/2$, $\gamma_2 \leq \gamma \leq \gamma_3$ and $F \leq V_2^p = \alpha(1 + \gamma)H/2 - (1 - \gamma + 2\alpha\gamma)L/2$.*

Similar to Lemma 3, Lemma 4 suggests that the firm develops the algorithm only when the algorithm is moderately reliable. Again, an unreliable algorithm does not help the firm update its pricing decision, while a highly reliable algorithm substantially reduces consumers' information rent and stifles search.

4.2 Imperfect Information

Now, consider the arguably more realistic scenario in which the firm's algorithm development decision is not observed by consumers. As consumers make their search decisions without knowing the firm's development decision, the model falls into games of imperfect information and we use a perfect Bayesian equilibrium as our solution concept.

Case 1. $\alpha H > L$.

Note first that when $\alpha H > L$, there always exists an equilibrium in which no consumers search and the firm does not develop the algorithm. In this equilibrium, consumers hold the belief that the firm does not develop the algorithm, and hence they expect that the firm will charge a price H to all consumer arrivals (which is rational given the belief). Therefore, consumers choose not to search. Consider now the firm. As no consumers search, the firm always makes zero profits from the consumers. Therefore, it does not want to waste money developing the algorithm, and thus consumers' belief is self-fulfilled. The above discussion is summarized in the following lemma.

Lemma 5 *Suppose that $\alpha H > L$ and consumers cannot observe the firm's development decision. Then, there always exists an equilibrium in which the firm does not develop the algorithm and consumers do not search.*

The above equilibrium creates an unappealing deadlock. Does there exist a more efficient equilibrium that breaks the deadlock? The following lemma shows that under certain circumstances, there does exist such an equilibrium.

Lemma 6 *Suppose that $\alpha H > L$ and consumers cannot observe the firm's development decision. There exists an equilibrium in which the firm develops the algorithm and consumers search if and only if $s \leq \alpha(1 - \gamma_1)(H - L)/2$, $\gamma_1 \leq \gamma \leq \gamma_3$, and $F \leq V_1 = ((1 + \gamma - 2\alpha\gamma)L - \alpha(1 - \gamma)H)/2$.*

Lemma 6 shows that when consumers' search cost and the cost of developing the algorithm are low enough and the reliability of algorithm is moderate, there exists an equilibrium in which the firm develops the algorithm and consumers search. Holding the belief that the firm develops a moderately reliable algorithm, all consumers expect a nonnegative surplus and are willing to search, i.e., $CS = -s + \alpha \Pr(I|H)(H - L) \geq 0$. The firm, anticipating all consumers will search, is also willing to develop the algorithm to reduce the asymmetry of information and improve its profit through price discrimination. Again, consumers' belief is self-fulfilling and the market functions well. Clearly, the equilibrium described in Lemma 6 Pareto dominates the equilibrium discussed in Lemma 5, and therefore is naturally selected in the case of multiple equilibria.

An interesting question is, does the firm always invest in the algorithm efficiently when its development decision is not observed by consumers? To answer this question, we compare the firm's development decision in Lemma 6 to that in the perfect information benchmark, and summarize the results in the following proposition.

Proposition 3 *Consider the case where $\alpha H > L$. When $s \leq \alpha(1 - \gamma_1)(H - L)/2$, $\gamma_1 \leq \gamma \leq \gamma_3$ and $V_1 < F < V_1^p$, the firm underinvests in the algorithm. Both firm profit and consumer surplus are lower in the case of underinvestment.*

Proposition 3 unveils an inefficiency associated with the firm's investment decision: when the firm's investment decision is not observed, under certain circumstances, the firm inefficiently

forgoes the algorithm: both the firm and consumers would be better off if the firm invests in the algorithm, yet in the unique equilibrium, the firm forgoes the investment opportunity and consumers do not search.

The rationale is as follows. Recall that when $\alpha H > L$, consumers are only willing to search when the firm develops an algorithm that is moderately reliable (i.e., $\gamma_1 \leq \gamma \leq \gamma_3$). In this case, the benefit of an algorithm is two-fold for the firm. First, the algorithm induces consumer search and a transaction value to the firm. Second, it reduces the asymmetry of information between the firm and the consumers, thereby improving the firms' profit through the price discrimination effect. Mathematically, we can decompose the value of the algorithm to the firm, V_1^p , in the following way:

$$V_1^p = \underbrace{\alpha H}_{\text{transaction value}} + \underbrace{\frac{\alpha(\gamma-1)H}{2} + \frac{(1+\gamma-2\alpha\gamma)L}{2}}_{\text{price discrimination value}}, \quad (4)$$

and the firm is willing to develop the algorithm as long as $F \leq V_1^p$.

When the firm's development decision is not observed by consumers, the consumers' search decision does not depend on the firm's development decision. Instead, it depends on consumers' belief of the firm's development decision: Consumers will search if and only if they believe that the firm develops an algorithm.

As a result, in the unobservable case, the firm's incentive to develop the algorithm is different. As the firm's actual development decision has no effect on consumers' belief, it does not affect the consumers' search decision either. As such, the transaction value in Equation (4) disappears. Now, the firm considers only the price discrimination effect and the value of the algorithm drops to

$$V_1 = \frac{\alpha(\gamma-1)H}{2} + \frac{(1+\gamma-2\alpha\gamma)L}{2} < V_1^p.$$

When $V_1 < F < V_1^p$, there is underinvestment. The firm opportunistically forgoes the algorithm so as to save the development cost. Anticipating this, consumers refrain from searching to begin with and exit the market.

An illustrative example. To better illustrate the above results, consider the following numerical example: $H = 3, L = 1, \alpha = 0.5, s = 0.2, \gamma = 0.5$ and $F = 1$. When the firm's development decision is observed by consumers, it develops the algorithm and makes a profit of $\pi^p = 1.5 + 0.125 - 1 = 0.625$, where 1.5 corresponds to the transaction value and 0.125 corresponds to the price discrimination effect. Consumers always search and their surplus is $CS^p = 0.05$. When the firm's development decision is not observed by consumers, however, the firm forgoes the algorithm and consumers do not search. Therefore, in equilibrium, both the firm and consumers receive zero payoffs. Clearly, the unobservability of the firm's development decision leads to suboptimal equilibrium outcomes.

Why does the firm forgo the algorithm in the unobservable case? To see this, assume for contradiction that there exists an equilibrium in which the firm develops the algorithm and all consumers search. The firm is therefore able to make a profit of $\pi = 0.625$ if it develops the

algorithm. Now suppose that the firm covertly deviates and does not develop the algorithm. Prior to search, consumers do not observe the firm's deviation and still hold the belief the firm develops the algorithm. As a result, all consumers follow the equilibrium strategy and conduct search. Given that all consumers visit the firm, the firm optimally charges a single price $p = H$ to all consumers and makes a profit of $\tilde{\pi} = \alpha H = 1.5 > \pi$. Therefore, the firm does have an incentive to deviate and the proposed equilibrium does not hold. In fact, the only equilibrium is that the firm forgoes the algorithm and consumers do not search.

It is worth mentioning that when $\alpha H > L$, consumer surplus can decrease with the cost of developing the algorithm, F . For example, if $H = 3, L = 1, \alpha = 0.5, s = 0.2$, and $\gamma = 0.5$, simple algebra suggests that consumer surplus is

$$CS = \begin{cases} 0.05 & \text{when } F \leq 0.625, \\ 0 & \text{when } F > 0.625. \end{cases}$$

and there is a discontinuous decrease in consumer surplus at $F = 0.625$. This result arises because a low development cost encourages the firm to invest in the (moderately reliable) algorithm, leaving high-type consumers a chance to be misclassified and underpriced. An increase in the development costs discourages the firm from developing the algorithm, thereby discouraging consumer search. The following proposition summarizes the result.

Proposition 4 *Suppose that $\alpha H > L$ and $s \leq \alpha(1 - \gamma_1)(H - L)/2$. There is a discontinuous decrease in consumer surplus at $F = V_1$.*

Case 2. $\alpha H \leq L$.

Next, consider the case in which the segment size of high-type consumers is small. We present all analysis in Appendix B and summarize the equilibrium outcome in the following lemma. In the case of multiple equilibria, we use Pareto dominance to select the dominant equilibrium.

Lemma 7 *Suppose that $\alpha H \leq L$ and consumers cannot observe the firm's development decision. There are three cases:*

1. *If $s > \alpha(H - L)$, the firm never develops the algorithm and consumers never search.*
2. *If $\alpha(1 - \gamma_2)(H - L)/2 < s \leq \alpha(H - L)$, the firm develops the algorithm with a positive probability if and only if $\gamma > \gamma_2$ and $F < V_2 = \alpha(1 + \gamma)H/2 - (1 - \gamma + 2\alpha\gamma)L/2$. In equilibrium, the firm develops the algorithm with probability λ and consumers search with probability ϕ , where λ and ϕ are given by:*

$$\alpha \left(1 - \lambda + \lambda \cdot \frac{1 - \gamma}{2} \right) (H - L) = s, \quad (5)$$

$$\phi \left(\frac{\alpha(1 + \gamma)H}{2} - \frac{(1 - \gamma + 2\alpha\gamma)L}{2} \right) = F. \quad (6)$$

3. If $s \leq \alpha(1 - \gamma_2)(H - L)/2$, the equilibrium outcome is as follows.

- (i) When $\gamma \leq \gamma_2$, the firm does not develop the algorithm and consumers search.
- (ii) When $\gamma_2 < \gamma \leq \gamma_3$, the firm develops the algorithm if and only if $F < V_2$. All consumers search.
- (iii) When $\gamma_3 < \gamma$, the firm develops the algorithm with a positive probability if and only if $F < V_2$. In equilibrium, the firm develops the algorithm with probability λ and consumers search with probability ϕ , where λ and ϕ are given by Equations (5) and (6).

Lemma 7 illustrates an interesting result: When the costs of search and algorithm development are low and the algorithm is reliable enough, both the firm and consumers adopt mixed strategies: The firm randomizes between developing the algorithm and not, and the consumers randomize between searching and not.

The reason is as follows. Suppose that all consumers search, i.e., $\phi = 1$. Given the volume of consumer arrivals, the firm strictly prefers to develop the algorithm to price discriminate amongst them. However, knowing that the firm always invests in the algorithm, consumers would prefer not to search, a contradiction. Alternatively, suppose that no consumers search, i.e., $\phi = 0$. In this case, the algorithm becomes useless and the firm would forgo it. However, knowing that the firm does not develop the algorithm, consumers would prefer to search and enjoy the low price, a contradiction. In sum, there only exists a mixed-strategy equilibrium in which the consumers randomize between searching and not, leaving the firm indifferent about whether or not to develop the algorithm. The firm also randomizes in its development decision, leaving consumers indifferent about whether or not to search.

When the firm's development in the algorithm is not observed by consumers, will the firm make its development decision efficiently? We compare Lemma 4 and Lemma 7 and summarize the results in the following proposition.

Proposition 5 *Consider the case where $\alpha H \leq L$. When (i) $\alpha(1 - \gamma_2)(H - L)/2 < s \leq \alpha(H - L)$, $\gamma > \gamma_2$ and $F < V_2$, or (ii) $s \leq \alpha(1 - \gamma_2)(H - L)/2$, $\gamma > \gamma_3$ and $F < V_2$, the firm overinvests in the algorithm. Both firm profit and consumer surplus are lower in the case of overinvestment.*

While Proposition 3 shows that the firm may underinvest in the algorithm when $\alpha H > L$, Proposition 5 shows that the opposite may be true when $\alpha H \leq L$: the firm may make an excessive investment in the algorithm. That is, the firm does not develop the algorithm when the investment is observed by consumers, but develops the algorithm with positive probabilities when the investment is not observed. Note that in the observed case, the algorithm has two effects on the firm's profit. First, it allows the firm to obtain a higher profit from consumer arrivals through price discrimination. Second, a reliable algorithm prevents consumers from searching. Mathematically,

the value of the algorithm is

$$V_2^p = \phi \left(\underbrace{\frac{\alpha(1+\gamma)H}{2} - \frac{(1-\gamma+2\alpha\gamma)L}{2}}_{\text{price discrimination effect on each consumer arrival}} \right) \quad (7)$$

where ϕ , the fraction of consumers that visit the firm, is dependent on the firm's development decision. A highly reliable algorithm drives ϕ to zero, and therefore the value of the algorithm vanishes and the firm does not develop the algorithm.

When the firm's development decision is not observable, the value of the algorithm is again given by Equation (7). The difference is, now, the fraction of consumers that visit the firm, ϕ , does not depend on the firm's development decision (which is not observed by consumers). Therefore, holding ϕ constant, the development of an algorithm only affects the firm's profit through the (positive) price discrimination effect, and the firm is willing to develop the algorithm even if it is highly reliable.

An illustrative example. Again, we use a simple example to illustrate the insight. Suppose that $H = 3, L = 2, \alpha = 0.5, s = 0.1, \gamma = 1$ and $F = 0.2$. In this example, the algorithm perfectly discerns high-type and low-type consumers. When the firm's development decision is observed by consumers, the firm will not develop the algorithm because the perfect algorithm will prevent consumers from searching, leaving zero profit to the firm. Therefore, in equilibrium, the firm does not develop the algorithm, offering a uniform price $p = L$ (since $\alpha H < L$), and all consumers search. The firm's equilibrium profit is $\pi^p = 2$, and consumer surplus is $CS^p = 0.4$, where the superscript p stands for perfect information.

When the firm's development decision is not observed by consumers, there is no pure strategy equilibrium. To show this, assume for contradiction that there exists an equilibrium in which consumers always search. Then it is optimal for the firm to invest in the algorithm and make a profit of $\pi = -F + \alpha H + (1 - \alpha)L = 2.3 > 2$. However, expecting that the firm will invest in the algorithm, consumers strictly prefer to not search, a contradiction. Next, assume for contradiction that there exists an equilibrium in which consumers never search. It is clearly optimal for the firm to not waste money on the algorithm. However, in anticipation of this, consumers strictly prefer to search to enjoy the low price $p = L$, a contradiction. In the mixed-strategy equilibrium, consumers search with probability $\phi = 0.4$, leaving the firm indifferent about developing the algorithm or not. The firm develops the algorithm with probability $\lambda = 0.8$, leaving consumers indifferent about whether or not to search. In equilibrium, the firm's profit is $\pi = 0.8$, a 60% loss over the perfect information benchmark. The expected consumer surplus is $CS = 0$. The total social welfare also suffers a striking 66.7% loss.

Interestingly, when overinvestment takes place, the firm profit can increase in the cost of developing the algorithm, F . For example, when $H = 3, L = 2, \alpha = 0.5, s = 0.1$ and $\gamma = 1$, the firm

profit is

$$\pi = \begin{cases} 4F & \text{when } F < 0.5, \\ 2 & \text{when } F \geq 0.5, \end{cases}$$

which (weakly) increases with F . This result arises because a higher development cost makes it more costly for the firm to use the algorithm, which makes it less likely for the firm to develop the algorithm (i.e., λ decreases in F). Following this logic, consumers are more willing to search (ϕ decreases with F) and the firm makes a higher profit. In the extreme case of $F = 0$ ($F > 2$), the development cost is so low (high) that the firm always (never) invests in the algorithm, and consumers never (always) search. We summarize the above discussion in the following proposition.

Proposition 6 *When underinvestment takes place, firm profit can increase in F .*

As information technology advances, the cost of developing a consumer algorithm falls dramatically. For example, the data storage cost for a gigabyte of a hard drive has decreased 75% from \$0.11 in 2009 to \$0.028 in 2017 (Klein, 2017; Li, 2018). Web innovators, such as Facebook, Google, and Yahoo, offer scalable storage and computing architecture to manage firms' consumer data, dramatically reducing the cost of data storage and management. It seems that a decrease in F only benefits the firm: If the firm develops the algorithm, it can save the development cost; if the firm does not develop the algorithm, its profit will not be affected. In either case, the firm is weakly better off with a lower F . However, Proposition 6 shows that this is not necessarily the case. A decrease in F incentivizes the firm to opportunistically develop the algorithm, even if it is not efficient to do so, which can backfire on the firm profit. Therefore, advances in information technology can lead to unintended consequences on firm profit.

5 Conclusion

Personalized pricing is becoming increasingly popular nowadays, and has gained attention from both marketing and operations management communities. Common wisdom holds that pricing algorithms reduce the asymmetry of information between the firm and consumers, which transfers consumer surplus into firm profits. We consider a novel model of personalized pricing under consumer search, and show that such "intuition" does not necessarily hold when consumers must incur a positive search cost to make an informed purchase from the firm.

We find that, first, a moderately reliable algorithm may benefit both the firm and consumers. This result arises because the algorithm leaves high-type consumers a decent chance to be misclassified (as low-type consumers) and thereby obtaining a better deal, which induces consumers to search. The firm benefits not only from the price discrimination effect, but also from the increased consumer search. On the other hand, a highly reliable algorithm can work to the detriments of both the firm and consumers. This result arises because a highly reliable algorithm allows the firm to effectively price discriminate amongst its consumers, substantially expropriating consumers' surplus. In anticipation, consumers choose to not visit the firm. As a result, there will

not be any sales, leaving zero payoffs to the firm and consumers, a clearly suboptimal outcome. Overall, the net effect of personalized pricing is *a priori* unclear; depending on the reliability of the algorithm, pricing algorithms can either aggravate or alleviate the holdup problem caused by consumer search. In Appendix C, we provide experimental evidence for our key model assumptions and results via Amazon Prime's M-Turk study.

Second, following the above logic, we find that both firm profit and consumer surplus can be non-monotone in the reliability of the algorithm. The firm's profit is maximized when the reliability of the algorithm is high but not too high. At this reliability, the firm is able to price discriminate amongst its consumers without deterring them from searching. In contrast, consumer surplus may peak when the reliability of the algorithm is low but not too low. At this reliability, the firm is willing to price discriminate amongst its consumers, yet the unreliable algorithm leaves consumers a high chance to be misclassified and therefore underpriced.

Third, we endogenize the firm's decision to develop the algorithm, assuming that the firm bears a fixed cost of developing the algorithm and that its development decision is not observed by consumers. We show that when the reliability of the profiling algorithm is moderate and the development cost is high, the firm underinvests in the algorithm. On the other hand, when the reliability of the pricing algorithm is high and the development cost is low, the firm overinvests in the algorithm. In either case, consumers are reluctant to search, leading to losses in firm profit and consumer surplus. Somewhat paradoxically, firm profit can be lower with an opportunity to develop the algorithm. Future research may examine the boundaries of this result in the presence of both product search and inspection costs.

Overall, this research contributes to our understanding of personalized pricing and consumer profiling, showing that a more reliable algorithm does not necessarily benefit a firm. Our research cautions firms and public policymakers that they must take consumer search into account when making relevant decisions and regulations. It also suggests researchers the importance of considering consumer search when studying personalized pricing. Lastly, we assume that consumers' valuation uncertainty is resolved when they arrive at the firm's location. Nonetheless, in certain cases consumers may have to incur an additional inspection cost to find out their true valuations even after arriving at the firm's location (Li and Xu, 2022). It would be of interest to explore firms' incentives to develop pricing algorithms in the presence of an inspection cost.

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Appendix

A Novelty

In this section, we discuss the novelty of our paper by illustrating the importance of studying both personalized pricing and consumer search.

A.1 The firm uses personalized pricing but consumers do not search

As in the paper, we assume that consumer valuations are either low or high and the firm receives a noisy signal of consumer valuations. We can show that the equilibrium firm profit is:

- If $\alpha H > L$:

$$\pi = \begin{cases} \alpha H & \text{when } \gamma < \gamma_1, \\ \frac{\alpha(1+\gamma)H}{2} + \frac{(1+\gamma-2\alpha\gamma)L}{2} & \text{otherwise,} \end{cases}$$

It follows immediately that the firm's profit increases with γ .

- If $\alpha H \leq L$:

$$\pi = \begin{cases} L & \text{when } \gamma \leq \gamma_2, \\ \frac{\alpha(1+\gamma)H}{2} + \frac{(1+\gamma-2\alpha\gamma)L}{2} & \text{otherwise,} \end{cases}$$

It follows immediately that the firm's profit increases with γ .

In sum, when consumers do not search, we find that the firm's profit always increases with γ , suggesting that a firm is always better off when the reliability of its algorithm improves.

A.2 The firm does not use personalized pricing but consumers search

As in the paper, we use s to denote consumers' search cost. We can find the equilibrium as follows:

- If $\alpha H > L$, no consumers search and the firm makes zero profit.
- If $\alpha H \leq L$ and $s \leq \alpha(H - L)$: all consumers search and the firm makes a profit of L .
- If $\alpha H \leq L$ and $s > \alpha(H - L)$: no consumers search and the firm makes zero profit.

Comparing the above results with that of the paper, we can see that personalized pricing can either improve or hurt the firm's profits, thereby generating additional insights.

B Proofs

Proof of Lemma 1. First, consumers never search when $\gamma < \gamma_1$. This is because, if a consumer searches, she will face a price $p_h = p_l = H$ and makes a total surplus of $-s < 0$. Now consider

the case in which $\gamma \geq \gamma_1$. A consumer's expected surplus when visiting the firm is

$$CS = -s + \alpha \Pr(l|H)(H - p_l) = -s + \alpha \cdot \frac{1 - \gamma}{2} \cdot (H - L).$$

Consumers search if and only if $CS \geq 0$. Solving the inequality proves the lemma. Q.E.D.

Proof of Lemma 2. The proof is analogous to that of Lemma 1. First, consider the case in which $\gamma \leq \gamma_2$. The firm offers a single price $p_h = p_l = L$ to all consumers, and a consumer obtains an expected surplus of $CS = -s + \alpha(H - L)$ when visiting the firm. A consumer is willing to search if and only if $s \leq \alpha(H - L)$. Next, consider the case in which $\gamma > \gamma_2$. The firm offers prices $p_h = H$ and $p_l = L$ to its consumers, and a consumer's expected surplus from visiting the firm is

$$CS = -s + \alpha \Pr(l|H)(H - p_l) = -s + \alpha \cdot \frac{1 - \gamma}{2} \cdot (H - L).$$

Note that when $s > \alpha(1 - \gamma_2)(H - L)/2$, consumer surplus is always negative. Otherwise, $CS \geq 0$ if and only if $\gamma \leq \gamma_3$. This completes the proof. Q.E.D.

Proof of Lemma 3. The firm is only willing to develop the algorithm if consumers are willing to search given the algorithm, i.e., when $s \leq \alpha(1 - \gamma_1)(H - L)/2$, $\gamma_1 \leq \gamma \leq \gamma_3$. When consumers are willing to search, the firm develops the algorithm when the cost of the development is smaller than the cost, i.e., when $F \leq V^p$. Q.E.D.

Proof of Lemma 4. The firm is only willing to develop the algorithm if consumers are willing to search given the algorithm, i.e., when $s \leq \alpha(1 - \gamma_2)(H - L)/2$, $\gamma_2 \leq \gamma \leq \gamma_3$. When consumers are willing to search, the firm develops the algorithm when the cost of the development is smaller than the cost, i.e., when $F \leq V^p$. Q.E.D.

Proof of Lemma 5. The proof follows immediately from the discussion. Q.E.D.

Proof of Lemma 6. Suppose that there exists an equilibrium in which consumers search with a positive probability $\phi > 0$. Then, sequential rationality requires that the firm's pricing strategy to be the one described in Section 3.1. Given the firm's equilibrium pricing strategy, consumers are only willing to search if and only if the firm invests in the algorithm and the reliability of the algorithm is moderate, i.e., $\gamma_1 \leq \gamma \leq \gamma_3$. Therefore, it suffices to consider the case in which $\gamma_1 \leq \gamma \leq \gamma_3$.

Now, consider the firm's optimal investment decision. If the firm does not invest in the algorithm, it makes a profit of $\pi_1 = \phi\alpha H$ (it sells only to high-type consumers who search). If the firm invests in the algorithm, it makes a profit of

$$\pi_2 = -F + \phi \left(\alpha \cdot \frac{1 + \gamma}{2} \cdot H + \alpha \cdot \frac{1 - \gamma}{2} \cdot L + (1 - \alpha) \cdot \frac{1 + \gamma}{2} \cdot L \right).$$

The firm invests in the algorithm if and only if $\pi_1 \leq \pi_2$. Straightforward calculation shows that there exists an equilibrium with $\phi = 1$ when $F \leq \bar{F}$. Moreover, all other equilibria are Pareto

dominated by the equilibrium described above. Q.E.D.

Proof of Lemma 7. Consider first the case in which $s > \alpha(H - L)$. It can be easily shown that regardless of the firm's investment decision, consumers cannot obtain a positive surplus from visiting the firm. Therefore consumers do not search. Given that consumers do not search, the firm does not invest in the algorithm.

Next consider the case in which $\alpha(1 - \gamma_2)(H - L)/2 < s \leq \alpha(H - L)$. Clearly, the firm does not invest in the algorithm when $\gamma \leq \gamma_2$ because such an unreliable algorithm does not help the firm fine-tune its pricing strategy at all. The case $\gamma > \gamma_2$ is more complex. Suppose that there exists an equilibrium in which consumers search with a positive probability $\phi > 0$. If the firm does not invest in the algorithm, the firm simply charges a price $p = L$ to all consumer arrivals and makes a profit of $\pi_1 = \phi L$. This leaves each consumer arrival an expected surplus of $CS_1 = -s + \alpha(H - L)$. If the firm invests in the algorithm, it optimally charges prices $p_h = H$ and $p_l = L$, making a total profit of

$$\pi_2 = -F + \phi \left(\alpha \cdot \frac{1 + \gamma}{2} \cdot H + \alpha \cdot \frac{1 - \gamma}{2} \cdot L + (1 - \alpha) \cdot \frac{1 + \gamma}{2} \cdot L \right).$$

leaving each consumer arrival an expected surplus of

$$CS_2 = -s + \alpha \cdot \frac{1 - \gamma}{2} (H - L).$$

Let λ be the probability that the firm invests in the algorithm, we have the following candidate equilibria where consumers search with positive probabilities:

- (1) $\lambda = 0$ and $\phi = 1$. The equilibrium is sustained iff $\pi_1 \geq \pi_2$ and $CS_1 \geq 0$.
- (2) $\lambda = 0$ and $\phi \in (0, 1)$. The equilibrium is sustained iff $\pi_1 \geq \pi_2$ and $CS_1 = 0$.
- (3) $\lambda = 1$ and $\phi = 1$. The equilibrium is sustained iff $\pi_2 \geq \pi_1$ and $CS_2 \geq 0$.
- (4) $\lambda = 1$ and $\phi \in (0, 1)$. The equilibrium is sustained iff $\pi_2 \geq \pi_1$ and $CS_2 = 0$.
- (5) $\lambda \in (0, 1)$ and $\phi = 1$. The equilibrium is sustained iff $\pi_1 = \pi_2$ and $(1 - \lambda)CS_1 + \lambda CS_2 \geq 0$.
- (6) $\lambda \in (0, 1)$ and $\phi \in (0, 1)$. The equilibrium is sustained iff $\pi_1 = \pi_2$ and $(1 - \lambda)CS_1 + \lambda CS_2 = 0$.

The proof follows by analyzing the above six cases. It can be seen that the only possible equilibrium falls into category (6), where both the firm and the consumers randomize their strategies.

Lastly, consider the case in which $s \leq \alpha(1 - \gamma_2)(H - L)/2$. Again, when (i) $\gamma \leq \gamma_2$, the firm does not invest in the algorithm because such an unreliable targeting technology does not help the firm fine-tune its pricing strategy at all. As for the other two cases, the analysis also involves discussing all the categories (1) to (6) listed above. When (ii) $\gamma_2 < \gamma \leq \gamma_3$, the equilibrium corresponds to category (3), where the firm always invests in the algorithm and consumers always search. When (iii) $\gamma_3 < \gamma$, the equilibrium corresponds to category (6), where both the firm and the consumers randomize their strategies. Q.E.D.

Proof of Proposition 1. First, when $s > \alpha(1 - \gamma_1)(H - L)/2$ or $\gamma < \gamma_1$ or $\gamma > \gamma_3$, consumers do not search and both firm profit and consumer surplus are zero. Otherwise, when consumers search, the firm charges a price $p_h = H$ to h -signal consumers and a price $p_l = L$ to l -signal consumers. Among the h -signal consumers, $\alpha(1 + \gamma)/2$ are high-valuation consumers that will make a purchase. In addition, there are $\alpha(1 + \gamma)/2 + (1 - \alpha)(1 + \gamma)/2 = (1 + \gamma - 2\alpha\gamma)/2$ l -signal consumers that purchase at p_l . In sum, the firm's profit is

$$\pi = \frac{\alpha(1 + \gamma)H}{2} + \frac{(1 + \gamma - 2\alpha\gamma)L}{2}.$$

As for consumers, only the misclassified high-valuation consumers obtain a positive surplus $H - L$ (net of the search cost). The size of this consumer segment is $\alpha(1 - \gamma)/2$, and the total consumer surplus is

$$CS = \frac{\alpha(1 - \gamma)(H - L)}{2} - s.$$

The remainder of the proof follows immediately. Q.E.D.

Proof of Proposition 2. The proof is analogous to that of Proposition 1 and is omitted. Q.E.D.

Proof of Proposition 3. The proof follows directly by comparing Lemmas 3 and 6. Q.E.D.

Proof of Proposition 4. The proof follows directly from the discussion. Q.E.D.

Proof of Proposition 5. The proof follows directly by comparing Lemmas 4 and 7. Q.E.D.

Proof of Proposition 6. The proof follows directly from the discussion. Q.E.D.

C Experimental Evidence

We conduct a within-subject experiment to test our main model assumptions and results. The goal of the experiment is twofold. First, to understand consumer preferences, we test the degree to which our modeling framework appears to capture the essence of how typical consumers react to stores with AI-based algorithmic software. Second, we test whether consumers would prefer to shop at a store that can understand their preferences near perfectly versus another store that is able to predict their preferences less accurately. Our study puts respondents in the shoes of online shoppers for apparel/backpacks where they are asked to imagine that they are thinking about buying a backpack. The survey was completed by $N = 251$ respondents (66.0% female) from the U.S. Each respondent earned a reward of \$1.50. The Web Appendix provides the complete survey used in this study along with the basic descriptives.

Our study begins as follows:

When you are shopping for apparel/backpacks, either online or in brick and mortar stores, retail companies now have the ability to be equipped with facial recognition and artificial intelligence software where firms can recognize customers, combine it with customers' purchasing habits

and preferences for their products and services based on shopper behavior, and steer their customers to the "right" product with the "right" price. When retail firms engage in such artificial intelligence analysis using data about customer shopping behavior, we refer to such analysis as "shopper analysis".

We then ask respondents (i) to what extent they think retail stores are currently using "shopper analysis" in general as well as to provide the right product at the right price to their consumers, and (ii) why they think retailers practice "shopper analysis." The results indicate that a majority of respondents (70.9%) agree that retailers are using some type of consumer data to arrive at prices for their products. Similarly, very few respondents (29.0%) said that retailers are not using any type of data for their pricing. This, combined with the 32.1% of respondents who thought that retailers are using AI software to recognize consumers and steer them to the right products and prices provides evidence that, from a consumer's perspective, while various types of retailers exist in the marketplace, consumers do think that retailers are using data-based analytics in their demand side operations.

We further investigate differences in perceptions of frequency of data-based analysis by retailers via an allocation of 100 points to three types of retailers: (i) those who do not use any type of consumer data, (ii) those who use some type of consumer data — but not a full-fledged AI-based shopper analysis, and (iii) those who use AI-based shopper analysis to come up with prices for their products. The results show that there was a significant difference in respondents' allocation of 100 points to each of the three types of retail stores ($p < .01$): those who do not use any type of consumer data (29.0), those who use some type of consumer data — but not a full-fledged AI-based analysis (38.8), and those who use AI-based shopper analysis (32.1) to come up with prices for their products. Further, more consumers thought retailers engage in AI-based shopper analysis because it helps them sell more profitable products to their consumers versus caring about their consumers (57.5% vs. 41.7%, $p = .049$). Thus, our survey results strongly support our key model assumption that consumers are quite aware that retailers do indeed practice data-based, algorithmic analysis to determine offers to consumers about their products and prices.

In order to understand how consumers would behave when faced with two retail stores, both implementing personalized pricing with one store being able to predict consumer preferences near perfectly and the other store not that perfectly, we gave the following information to the respondents:

Imagine, you are thinking about buying a backpack. You are willing to pay \$150 for a black backpack and \$100 for a white backpack. If the price is above \$150 for a black backpack or above \$100 for a white backpack, you will not buy it. You know two stores, A and B (both either online or brick and mortar stores near you), that sell backpacks. You are also aware that both Stores, A and B (either online or brick and mortar stores near you), use an artificial intelligence based Shopper Analysis capability that tries to understand your preferences.

So, you know that the two stores, A and B, sell backpacks. However, you do not know the color

and the price of backpacks that each store's shopper analysis could offer you. If you visit a store (either on their online website or take a short walk nearby), each store's shopper analysis tries to predict your preferences for the two colors of backpacks and your willingness to pay for black and white backpacks. Accordingly, based on its understanding of your preferences, each store's shopper analysis will offer you one of the following four combinations of prices and color of backpack. The chance of you seeing each option and the details of each option in the two stores are as follows (note that the probabilities all add up to 100% in each store):

Store A			Store B		
Option	Color	Chance of prices	Option	Color	Chance of prices
1	Black	40% chance price is \$150	1	Black	49% chance price is \$150
2	Black	10% chance price is \$100	2	Black	1% chance price is \$100
3	White	40% chance price is \$100	3	White	49% chance price is \$100
4	White	10% chance price is \$150	4	White	1% chance price is \$150

Table 3: Store Choice Question

We asked respondents: How likely would you be to shop at Store A versus Store B? Please use a 0-100 scale, where 0 indicates "Definitely shop at Store A" and 100 indicates "Definitely shop at Store B". Consumers felt that they are more likely to shop at Store A (Store A $mean = 57.11$ vs. 42.89 for Store B, $SD = 34.16$; $p < 0.01$). This suggests that, in line with our model prediction, consumers would prefer to shop at a retail store that has a less than perfect algorithm relative to a store that has a near-perfect algorithm in terms of being able to predict their preferences.

We also ask participants if they find the scenarios presented in the study were realistic (1=Not all Realistic; 7=Extremely Realistic): the mean score was 5.87 ($SD = 1.13$). A similar 7-point scale showed that the participants also found the study interesting ($mean = 5.03$, $SD = 1.61$). Overall, our survey results indicate that (a) consumers are quite aware of retailers using data-based algorithmic software, a factor that is incorporated into our modeling environment, and (b) consumers would rather prefer to shop at a store that is more imperfect in its ability to properly predict consumer preferences relative to a store that is more perfect. Note that while our analytical model attests to the fact that consumers do not prefer algorithms that can perfectly predict consumers' preferences, it still leaves open the question of the external validity of our result. The experimental results we show above that consumers prefer less than perfect personalized pricing speaks to the external validity of our analytical result.