Transparency of Behavior-Based Pricing

Behavior-based pricing (BBP) refers to the practice in which firms collect consumers' purchase history data, recognize repeat and new consumers from the data, and offer them different prices. BBP is a prevalent practice for firms and a worldwide concern for consumers. Extant research has examined BBP under the assumption that consumers observe firms' practice of BBP. However, consumers do not know this for specific firms and are often unaware of how firms collect and use their data. In this paper, we examine how firms make BBP decisions when consumers do not observe whether firms perform BBP and how the transparency of firms' BBP practice affects firms and consumers. We find that when consumers do not observe firms' practice of BBP and the cost of implementing BBP is low, a firm indeed practices BBP, even though BBP is a dominated strategy when consumers observe it. When the cost is moderate, the firm does not use BBP; however, it must distort its first-period price downward to signal and convince consumers of its choice. A high cost of implementing BBP serves as a commitment device that the firm will forfeit BBP, thereby improving firm profit. By comparing regimes in which consumers observe and do not observe a firm's practice of BBP, we find that transparency of BBP increases firm profit but decreases consumer surplus and social welfare. Therefore, commanding firms to disclose collection and usage of consumer data could hurt consumers and lead to unintended consequences.

Key words: behavior-based pricing, privacy, transparency, signaling, game theory

In the era of big data, firms across a wide range of industries use technologies such as Internet cookies, click-stream information, loyalty cards, and automatic data-gathering devices to collect consumers' data (Bread 2016). Firms use consumers' purchase history data to practice behavior-based pricing (BBP); that is, firms recognize repeat and new customers from their purchase history data and charge them different prices. Firms' practice of BBP is prevalent in a wide range of industries. However, despite this prevalence, consumers are often unaware of whether a particular firm collects and uses their information to price discriminate them. Two factors contribute to consumers' lack of awareness. First, they do not observe a particular firm's investment in data-collecting infrastructures and decision to practice BBP (Acquisti, Brandimarte, and Loewenstein 2015). Therefore, although consumers may know that many firms collect and use consumer data, they cannot detect whether the particular firm they interact with collects and uses their data for BBP. Second, firms typically do not clearly communicate to consumers how they collect, use, and exploit their data (Miller 2014). Although "some companies are open about their data practices, most prefer to keep consumers in the dark, choose control over sharing, and ask for forgiveness rather than permission" (Morey, Forbath, and Schoop 2015). Indeed, a Federal Trade Commission study found that 80% of randomly sampled websites that collect consumer information do not provide clear and conspicuous notice of their information collection or explicitly ask for consumers' consent (FindLaw 2018).

Firms' secret collection and usage of consumers' data for BBP have generated considerable concerns from consumers and legislative authorities (e.g., Goldfarb and Tucker 2011). In the United States, personal data laws are not clearly defined or enforced by any independent governing body, and thus debate about the legality of BBP is ongoing. In 1996, a consumer living in Manhattan sued Victoria's Secret for distributing different versions of catalogs with identical items but different prices. However, the New York Court dismissed the claim by noting that it was an accepted business practice to reward repeat consumers or to draw in new consumers with special savings (Miller 1996). Any form of price discrimination is legal in the United States, as long as the basis of discrimination is not race, religion, national origin, gender, and the like (Ramasastry 2005).

Although the practice of BBP is legal, consumer advocates call for regulations that mandate that firms at least disclose the practice of BBP to consumers (Weiss and Mehrotra 2001, Miller 2014). The goal is to ensure that consumers are aware of firms' collection and usage of their purchase history data for future price discrimination against them. With this information, consumers can choose whether to purchase from such firms and give away their data. For this purpose, in 2012, the Ensuring Shoppers Transparency in Online Pricing (E-STOP) Act[1] proposed rules that require Internet merchants to disclose whether they use consumers' personal information for price discrimination (Li and Jain 2016). The White House has issued

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[1] H.R. 6508, 112th Cong., 2nd sess. (September 21, 2012).

several Big Data reports about firms' BBP practice since 2014. Nevertheless, it is inconclusive whether regulations should be introduced to require disclosure of BBP practice to consumers (InsidePrivacy 2015). Extant research offers little guidance for public policy makers in undertaking this important regulation, because it has mainly examined BBP under the assumption that consumers can directly observe firms' practice of BBP. Research has not investigated how firms make BBP and pricing decisions when consumers do not observe this practice. Therefore, it is unclear how transparency of BBP affects firms, consumers, and social welfare.

The objective of this article is to fill these research gaps. Specifically, we address the following research questions: (1) When consumers do not observe a firm's practice of BBP, how does the firm decide whether to use BBP and set prices? and (2) How does transparency of the firm's BBP practice affect firm profit, consumer surplus, and social welfare? Answers to these questions can guide firms in making investment and pricing decisions about BBP when consumers cannot observe this practice. Our findings also provide important guidance for public policy makers charged with regulating BBP transparency to protect consumer privacy and welfare.

To address these questions, we consider a three-period game-theoretic model in which a monopolist firm first decides whether to incur a fixed cost F to implement BBP in period 0. Then, the firm sells a repeatedly purchased product to consumers in periods 1 and 2. We examine a benchmark model in which consumers observe the firm's practice of BBP (i.e., a perfect-information regime with transparency of BBP) and a main model in which consumers do not observe this practice (i.e., an imperfect-information regime without transparency of BBP). Under imperfect information, consumers form beliefs about the firm's usage of BBP based on observed first-period price. In this regime, we examine the firm's BBP usage and pricing decisions. Finally, we compare the two regimes with and without BBP transparency to evaluate its effects on firm profit, consumer surplus, and social welfare.

Our analysis yields the following findings. First, research has established that when consumers observe firms' practice of BBP, BBP leads to lower profits (Fudenberg and Tirole 2000, Fudenberg and Villas-Boas 2006). Therefore, firms do not invest in data collection for BBP in period 0. By contrast, we find that when consumers do not observe the firm's practice of BBP, the firm may choose to invest in data collection for BBP in period 0. Specifically, the firm invests in BBP when the cost of implementation is low. In this case, BBP enables the firm to improve second-period profit by price discriminating between previous and new consumers. Given that consumers do not observe the firm's investment in BBP, such investment does not affect their purchase decision or firm profit in the first period. Therefore, when the cost of implementing BBP is low, using BBP increases the second-period profit: the firm guarantees a higher second-period profit through price discrimination without affecting its first-period profit. In equilibrium, strategic consumers

anticipate that the firm has an incentive to practice BBP in the second period. Consumers form the belief that the firm will practice BBP and make first-period purchase decisions accordingly. As a result, the firm's total profit over two periods ends up being lower than its profit under transparency without BBP. Despite this, when consumers cannot observe its practice of BBP, the firm cannot credibly commit to not practicing BBP.

Second, we find that when the cost of implementing BBP is moderate, the firm does not invest in BBP but needs to distort its first-period price downward to signal to consumers that it does not practice BBP. Such price distortion renders the second-period practice of BBP unprofitable, thereby convincing consumers that it does not practice BBP. This price distortion is unnecessary when the cost of implementing BBP is high, because a high cost sends a credible signal to consumers that the firm will not make a profit from using BBP. Therefore, a high cost serves as a commitment device that the firm will not deviate to invest in BBP.

Third, when consumers do not observe a firm's practice of BBP, the cost of implementation has a non-monotone impact on firm profit. When the cost is low, the firm incurs the cost to implement BBP, and its profit decreases with the cost. When the cost is moderate or high, a lower cost of implementing BBP makes investment more profitable, leading consumers to question whether the firm invests in BBP. As such, the firm needs to distort its first-period price more to convince consumers that it does not invest in BBP, and its profit declines accordingly. In addition, given that downward price distortion enables more consumers to buy the product, consumer surplus and social welfare increase when the firm distorts its price downward. These results imply that as information and data technology advance, the decrease in the cost of implementing BBP can hurt firms and benefit consumers and society as a whole.

Fourth, our key research question pertains to the impact of BBP transparency on firms and consumers. Comparison of the equilibrium outcomes with and without transparency of BBP shows that transparency increases firm profit but decreases consumer surplus and social welfare. This is because BBP transparency enables a firm to credibly commit to not practicing BBP without distorting its price downward. Therefore, according to our model, regulations that mandate that firms disclose their collection and usage of consumers' data for BBP are actually beneficial to firms and detrimental to consumers and society. Thus, our research cautions public policy makers that regulations designed to protect consumer privacy and welfare can lead to unintended consequences.

Last, we extend the main model to examine the case when the firm has had previous transactions with customers and thus can offer them personalized enhanced service. We find that the firm's ability to offer such service reduces its profit when the cost of implementing BBP is high but not too high. The rationale is that the firm's ability to offer personalized enhanced service improves the profitability of BBP. As a result, when the cost of implementing BBP is high such that the firm chooses not to practice BBP, convincing consumers

of its choice becomes increasingly costly. Therefore, the firm's ability to offer enhanced service decreases its profit. We also show how our main intuition continues to hold in competitive settings, in situations when consumers use anonymizing technologies to hide their identity or do not observe the cost of implementing BBP and in dynamic settings with endogenous choice of transparency.

RELATED LITERATURE

This article is closely related to the stream of research on BBP (Fudenberg and Villas-Boas 2006). A series of studies has established that when consumers observe whether firms invest in BBP, BBP reduces firm profits. Therefore, firms should not practice BBP even when practicing BBP is costless. This conclusion applies to both monopoly and duopoly settings. BBP reduces a monopolist's profit because of the *ratchet effect*: knowing that a firm uses BBP, consumers understand that it will use their first-period purchase decisions to price discriminate against them in the second period. Strategic consumers have incentives to postpone purchase to enjoy the low price for new consumers in the second period. Thus, the firm must reduce first-period prices to induce strategic consumers to buy in the first period. The firm's profit declines accordingly and is lower than when it does not invest in BBP and charges a single price (Villas-Boas 2004). Thus, although it is feasible for a monopolist to use BBP, it will never find it optimal to do so (Acquisti and Varian 2005). Fudenberg and Tirole (2000) use a two-period model to illustrate that BBP is also unprofitable for competing firms, though the mechanism is different from that in a monopoly. In particular, BBP leads competing firms to poach each other's customers. Competition in the second period becomes more intense and firms' total profits decline from the level without BBP. Villas-Boas (1999) draws the same conclusion by analyzing overlapping generations of consumers in an infinite-period game. Zhang (2011) further shows that when firms customize the horizontal attributes of products, profits become even lower than when firms only practice BBP. Research has also found some contexts in which firms can profit from BBP, such as when consumers have heterogeneous demand and preferences change over time (Shin and Sudhir 2010), asymmetric firms determine product quality (Jing 2017), consumers care about price fairness (Li and Jain 2016), competing products are vertically differentiated (Rhee and Thomadsen 2017), both manufacturers and retailers use BBP (Li 2018), or consumers are sufficiently averse to loss on match quality (Amaldoss and He 2018).

This article differs from the rich body of research on BBP in three important respects: First, prior studies assume that firms' practice of BBP is transparent and consumers can observe whether they implement BBP or not. By contrast, our research examines situations when consumers cannot observe whether firms practice BBP. Second, we compare situations when consumers observe and do not observe BBP to assess how transparency of BBP affects firms and consumers. Third, in past research the cost of BBP implementation

did not play a major role in the practice of BBP. By contrast, our model allows for a cost for investment in data infrastructure and collection for BBP. The treatment of costly BBP is consistent with the notion that "firms are making massive investments into building information infrastructures that allow them to collect, store, and analyze consumer data" (Acquisti and Varian 2005). Pazgal and Soberman (2008) also consider a cost of implementing BBP and firms incur the cost when they create additional benefits to repeat customers.

This article considers situations in which consumers do not observe whether a firm practices BBP; thus, the firm signals its (unobserved) BBP decision to consumers through its (observed) price decision. This type of game differs from exogenous signaling games in that the firm's private type is its *endogenous* decision; we refer to this as the *endogenous signaling game* (In and Wright 2018). Endogenous signaling games are most often assessed in the context of quality signaling, in which the product quality is firms' private choice. Klein and Leffler (1981) investigate a model in which firms decide their product qualities which are not observed by consumers, and show how firms can use price to signal their quality. Wolinsky (1983) considers a similar signaling game but assumes that consumers have noisy information about the true quality of the products. Bester (1998) finds that imperfect information about the product quality can reduce competing firms' incentives for horizontal product differentiation. Endogenous signaling games have been used in other contexts as well. For example, Rao and Syam (2001) analyze two competing supermarkets each selling two goods. Each supermarket only advertises the price of one good, and consumers infer the price of the unadvertised good from the price of the advertised good. Li, Rocheteau, and Weill (2012) examine the liquidity of assets in trade, in which agents choose a portfolio of genuine and fraudulent assets for trade and the terms of trade. Xu and Dukes (2017) consider consumers who are uncertain about their type and a monopolist who possesses superior information on consumer preferences through data aggregation technologies. Like in this paper, the firm incurs a signaling cost to convince consumers that they are not being taken advantage of by the monopolist. As a result, the monopolist cannot extract all consumer surplus even with price discrimination. Xu and Dukes (2019) examine a monopolist's product line design decisions when consumers exhibit perceptual errors when assessing their intrinsic preferences and firms use aggregate consumer data to infer consumer preferences. They show that consumers' rational suspicions may prevent the firm from exploiting its information advantage and the firm may offer efficient product quality to convince consumers that they are not being overcharged. Our research shows a similar signaling cost when the firm distorts its first-period price to convince consumers that it will not perform BBP. Different from these studies, we consider situations when consumers know their own preferences but firms can partially learn about consumers' preferences from their purchase history data. Furthermore, we examine how transparency of firms' BBP practice affects firms, consumer surplus, and social welfare.

In many situations, researchers find that imperfect observability can trigger opportunistic behavior, which yields inefficient equilibrium outcomes. In a one manufacturer, two retailers setup, Hart and Tirole (1990), O'Brien and Shaffer (1992), and McAfee and Schwartz (1994) all discover that, when a retailer cannot observe the contract terms between the manufacturer and the rival retailer, the manufacturer has an incentive to opportunistically renegotiate another's contract to increase bilateral profit at the retailer's expense. As a result, downstream competition becomes too fierce, and the manufacturer cannot achieve the first-best outcome. Coughlan and Wernerfelt (1989) show that, under supply chain competition, when the structure of one supply chain is not observed by the rival chain, strategic decentralization never occurs as an equilibrium outcome, in contrast with the finding of McGuire and Staelin (1983). Janssen and Shelegia (2015) find that, when consumers are uninformed about the wholesale prices manufacturers charge to retailers, the equilibrium prices become inefficiently high, which worsens double marginalization and lowers manufacturers' profits. Ben-Porath, Dekel, and Lipman (2018) show that the unobservability of project choice can induce firm managers to choose riskier and less profitable projects. Roy, Gilbert, and Lai (2018) reveal that in a distribution channel, when the manufacturer cannot observe the retailer's inventory level, the retailer will opportunistically overstock, which backfires on its own profit.

In this article, we examine the effect of unobservability in the context of BBP. While the literature on unobservability assumes that the firm is free to make its (unobserved) choices, we assume that the firm must incur a cost to practice BBP. The unobservability of the BBP decision also leads to the firm's opportunistic behavior: it cannot help opportunistically practicing BBP when the BBP implementation cost is low. Moreover, when the BBP implementation cost is moderate, the firm does not practice it; however, it must distort its price to convince consumers of its choice. Such distortion has not been covered in the literature.

This research is also related to studies on consumer privacy and transparency of firm decisions. For example, Rossi and Chintagunta (2016) find that posting fuel price signs (i.e., transparency of prices) decreases price levels without affecting price dispersions. Tucker (2014) empirically shows that when consumers have control over how their personally identifiable information to be used to personalize ads, they become nearly twice more likely to click on personalized ads. Our article supplements these studies by examining how transparency of firm's BBP practice affects prices, firm profit, consumer surplus, and social welfare.

Given that this paper is pertaining to cost frictions and commitment issues for dynamic pricing, this research is also related to the literature on the Coase conjecture and revenue management when firms sell durable goods. Coase (1972) shows that when a monopolist sells a durable good over two periods, consumers anticipate that prices will drop in the second period and withhold purchase in the first period. To induce consumers to buy in the first period, the monopolist reduces its price in the first period, resulting in lower total profits over two periods. Dilmé and Li (2019) examine a dynamic model when a monopolist

serves consumers who privately arrive and face an attention cost if they wait for future flash sales. They find that it is possible for the monopolist to sporadically hold flash sales to lower the stock of goods even though there may be high valuation buyers who arrive later. This results in another channel of inefficiency. Oery (2016) considers an online monopolist that uses targeted advertising to lure customers who looked but did not buy. The analysis shows how advertising costs can serve as a commitment device and higher advertising costs can actually benefit firms. Our research differs from this stream of literature in that we consider repeatedly-purchased goods that consumers buy in each period instead of durable goods that consumers only need to buy in one of the two periods. From consumers' purchase decisions in the first period, firms infer consumers' preferences and offer different prices on the basis of consumers' purchase histories.

THE MODEL

To assess how BBP transparency affects firms and consumers, we consider two regimes: with and without transparency of BBP. The first regime reflects situations when regulations require the firm to disclose its choice of BBP (i.e., $s \in \{\mathsf{BBP}, \emptyset\}$) to consumers. In this case, consumers have perfect information about whether the firm practices BBP. In the second regime, no regulations require the firm to disclose its BBP choice to consumers. Without the government's oversight, the firm cannot credibly disclose its BBP choice to consumers. Therefore, consumers have imperfect information about whether the firm practices BBP.

The model consists of three periods, $t = 0, 1, 2$. A monopolist firm sells a repeatedly-purchased product in the market. The unit cost to produce the product is constant, and we normalize it to zero. To simplify analysis and exposition, we assume that both the firm and the consumers are risk neutral and do not discount the future.

Information Collection and Behavior-Based Pricing

Technology infrastructures such as customer relationship management systems and data storage and management allow the firm to collect consumer information, practice BBP, and price discriminate between consumers who bought and did not buy in a previous period. At $t = 0$, the firm chooses whether or not to incur a cost F to invest in these data-collecting technologies for BBP. If the firm invests in BBP, it collects consumer information that allows it to classify consumers as either "previous" or "new" consumers. Subsequently, the firm is able to price discriminate against consumers on the basis of their purchase histories. Alternatively, if the firm chooses not to invest in BBP (denoted by $\emptyset$), it cannot collect consumers' information or price discriminate against them.[2] Let $s \in \{\mathsf{BBP}, \emptyset\}$ denote the firm's period-0 choice of whether or not to invest in BBP.

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[2] Our result continues to hold even if the firm can make a later investment in BBP at the beginning of period 2.

For the moment, we assume that the firm can only use consumer information to price discriminate against consumers. Later, we extend the model by considering the case when the firm can also offer personalized enhanced service to previous consumers.

Consumers

Consider now the demand side. There is a continuum of consumers with total mass normalized to 1. Each consumer has unit demand for the product at $t = 1$ and $t = 2$. Consumer i's valuation for the product is constant over time and uniformly distributed over the unit interval (i.e., $v_i \sim \mathsf{U}[0,1]$).

We assume that all consumers are sophisticated: they understand how their purchase behavior will affect their future prices and take it into account when making their purchase decisions. In particular, consumers understand that when the firm uses BBP, buying early may not be the best strategy because doing so reveals higher preferences to the firm, which induces the firm to charge them a higher price in the future. We also assume that while consumers cannot observe the BBP decision, they can observe the implementation cost F. In practice, consumers may develop a sense of the BBP implementation cost through media coverage and industry and government reports. Nonetheless, consumers may not always observe the cost F perfectly. In the discussion section on page 26, we discuss how our main insights continue to hold when consumers do not observe the cost F.

Timing and Decisions

The game unfolds in three periods. At $t = 0$, the firm decides whether or not to invest in BBP (i.e., $s \in \{\mathsf{BBP}, \emptyset\}$). If the firm invests in BBP ($s = \mathsf{BBP}$), it incurs the implementation cost F. At $t = 1$, the firm has no specific information about individual consumers and thus offers a single price p_1 to all consumers whether or not it practices BBP. After sales commence, the firm cannot practice BBP if it failed to make an investment earlier. Actions at $t = 2$ depend on the firm's choice of s. If $s = \mathsf{BBP}$, the firm offers two prices to the two identified groups: a price p_2^r to all previous consumers who purchased the product at $t = 1$ and a price p_2^n to all new consumers who did not purchase the product. If $s = \emptyset$, the firm again offers a single price p_2 to all consumers. Figure 1 summarizes the sequence of events.

Benchmark Regime: With Transparency of BBP

When a firm is required to give consumers clear and conspicuous privacy notices whenever it collects and uses consumer data for BBP, the firm's BBP decision becomes public information in the market. Knowing that the firm does not collect their information, consumers need no longer fear being price discriminated against by the firm in the future. We first analyze the transparency regime, in which consumers perfectly
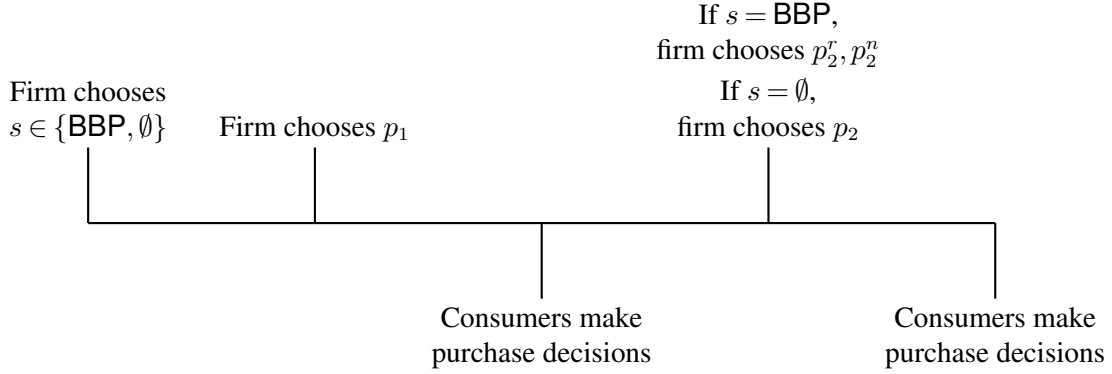
Figure 1   Sequence of events

observe the firm's choice $s \in \{\mathsf{BBP}, \emptyset\}$ after it is made. We solve the model using backward induction and state results in Lemma 1 and Table 1.

LEMMA 1. *With transparency of BBP (i.e., under perfect information), it is optimal for the firm not to practice BBP (i.e., the optimal s is $\emptyset$).*

Table 1   Optimal decisions and maximal profit under perfect information

	p_1	p_2	π_1	π_2	π
$s = \emptyset$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
$s = \mathsf{BBP}$	$\frac{3}{10}$	$p_2^r = \frac{3}{5},\ p_2^n = \frac{3}{10}$	$\frac{3}{25}$	$\frac{33}{100}$	$\frac{9}{20} - F$

The results in Table 1 indicate that, under perfect information, the firm is always worse off practicing BBP (with at least 10% loss of profit). The intuition is as follows: with BBP, when a consumer buys at $t = 1$, the firm knows that this consumer has a high valuation and thus charges him or her a higher price at $t = 2$. This *price discrimination effect* increases the firm's period-2 profit. However, fearing that the firm will price discriminate against them, some consumers defer their purchases until $t = 2$. Anticipating this reluctance to purchase, the firm must offer a lower initial price to induce period-1 purchases. This negative *ratchet effect* yields a profit loss for the firm. In line with previous BBP research (e.g., Villas-Boas 2004, Acquisti and Varian 2005), in our perfect-information setup, the ratchet effect dominates the price discrimination effect, and it is in the best interest of the firm *not* to practice BBP.

As Table 1 shows, if $s = \mathsf{BBP}$, the firm's profit decreases with F. Alternatively, if $s = \emptyset$, the firm's profit does not change with F. Because the firm always chooses $s = \emptyset$, F has no effect on the firm's profit.

MAIN REGIME: WITHOUT TRANSPARENCY OF BBP

In this section, we consider the main regime when consumers do not observe whether the firm invests in BBP. We discuss the information structure and the solution concept and then solve the equilibrium.

Information Structure and Solution Concept

Because consumers do not observe the firm's choice of s, the model falls into a game of imperfect information. We solve the game using the solution concept of perfect Bayesian equilibrium (PBE), under which consumers hold a belief about what the firm has chosen and act optimally given their belief.

Note that our model differs from exogeneous signaling games because within our model, it is the firm, not nature, that makes the choice $s \in \{\mathsf{BBP}, \emptyset\}$. This class of game is also known as the *endogenous signaling game* (In and Wright 2018), in which a sender (the firm) "signals" its private choice to receivers (consumers). As PBE does not impose any restrictions on beliefs off the equilibrium paths, the endogenous signaling games normally suffer from a plethora of equilibria. To focus on the most reasonable equilibria, we resort to *reordering invariance* (RI) as our refinement criterion. RI was initially proposed in In and Wright (2018) and has been widely used in the literature (e.g., Lester, Postlewaite, and Wright 2012, Li, Rocheteau, and Weill 2012, Rhodes 2014).

As described previously in the game, the firm first makes its unobserved choice $s \in \{\mathsf{BBP}, \emptyset\}$ and then makes the observed choice p_1. According to RI, a reasonable PBE should also be an equilibrium when we reverse these two decisions. For the current game, a reasonable equilibrium should also hold when the firm first makes the observed choice p_1 and then makes the unobserved choice $s \in \{\mathsf{BBP}, \emptyset\}$. RI has an intuitive appeal. According to In and Wright (2018), if the firm chooses the (observed) period-1 price p_1 before making the (unobserved) BBP decision s, then a subgame starts from the choice of p_1. We could then apply subgame perfection to discover consumers' belief about s, which should be optimal given p_1. Now, suppose instead that the BBP decision s is made first, there is no proper subgame (except for the whole game), and PBE itself does not impose any restrictions on beliefs off the equilibrium path. As such, there will be a large number of equilibria arising from various specifications of the consumers' out-of-equilibrium beliefs. Nevertheless, as the firm makes its BBP decision s and price decision p_1 without gaining any information in between, the order in which it makes the two decisions should not matter. Intuitively, even if the BBP decision is made before the price decision, the firm should already have in mind the price it is going to set. Following this logic, a rational firm should make the same choices for s and p_1 no matter which decision it makes first. For consumers, they are aware that the firm's BBP decision s should be optimally chosen given the price p_1. As a result, they can take the observed price p_1 as if it was chosen first.

In compliance with RI, in our analysis, we consider the reordered game in which the firm first chooses its first-period price p_1 and then decides whether to practice BBP.[3] Because the firm's choice of s is private, we must distinguish the firm's actual choice, s, from consumers' conjecture about that choice, which we illustrate as follows: after observing p_1, consumers form their belief of $s = \mathsf{BBP}$, which is denoted by $\Lambda(p_1) \in [0,1]$. That is, if $\Lambda(p_1) = 1$, consumers believe that the firm always practices BBP; if $\Lambda(p_1) = 0$, consumers believe that the firm does not practice BBP. If $\Lambda(p_1)$ is in between, consumers believe that the firm randomizes its choice between BBP and $\emptyset$. If multiple equilibria survive the RI-refinement, we select the equilibrium that has the lowest consumer belief of the firm having implemented BBP. We use this equilibrium selection criterion for a few reasons. First, this equilibrium selection criterion maximizes the firm's profit. Second, when forward induction applies, this equilibrium selection criterion picks the same equilibrium as what the forward induction criterion picks.

Analysis

We now solve for the equilibrium without transparency of BBP (i.e., under imperfect information). With RI-refinement, it suffices to consider the reordered game in which the firm first makes the observed decision on p_1 and then makes the unobserved decision on s. The solution entails first solving for the subgame following the choice of p_1. We then use the results to derive the firm's optimal choice of p_1. To break ties, we assume that when the firm is indifferent about whether or not to practice BBP, it does not practice BBP. We relegate the detailed analysis to the Appendix and present the equilibrium outcomes in Proposition 1 and Table 2.

PROPOSITION 1. *Without transparency of BBP (i.e., under imperfect information), the firm practices BBP when $F < \frac{7-2\sqrt{10}}{45} \approx 0.015$.*

Table 2   Equilibrium Strategies under Imperfect Information

	cost	s	p_1	p_2	π_1	π_2	π
$F < \frac{7-2\sqrt{10}}{45}$	low	BBP	$\frac{3}{10}$	$p_2^r = \frac{3}{5}$, $p_2^n = \frac{3}{10}$	$\frac{3}{25}$	$\frac{33}{100}$	$\frac{9}{20} - F$
$\frac{7-2\sqrt{10}}{45} \leq F \leq \frac{1}{16}$	medium	$\emptyset$	$2\sqrt{F}$	$\frac{1}{2}$	$2\sqrt{F} - 4F$	$\frac{1}{4}$	$2\sqrt{F} - 4F + \frac{1}{4}$
$\frac{1}{16} < F$	high	$\emptyset$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

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[3] In the reordered game, the firm still makes its BBP decision before the period-1 sale commences. Therefore, the firm's BBP decision does not hinge on consumers' period-1 purchase behavior.

When the firm's choice of BBP is transparent, the cost of implementing BBP does not play a major role (Fudenberg and Tirole 2000, Esteves 2009). This is because when consumers observe whether a firm practices BBP, the firm forfeits BBP in period 0 even if implementing BBP is costless. Given that firms do not implement BBP, the cost of implementing BBP has no impact on firm profit. By contrast, Proposition 1 reveals that when consumers do not observe whether a firm practices BBP, the cost of implementing BBP affects the firm's BBP and pricing decisions. We also show that the impact of F on firm profit, consumer surplus, and social welfare is nonmonotone.

F **is low: endogenous choice of BBP.** In contrast with the prediction that firms do not practice BBP under the perfect-information assumption, Proposition 1 shows that when F is low and consumers cannot observe the firm's choice of BBP, BBP becomes the firm's equilibrium choice. This result is relevant because new technology has significantly reduced the cost of data storage and management for implementing BBP. Thus, this result suggests that as the cost of implementing BBP continues to decline over time, we expect to witness growing practice of BBP.

To illustrate the intuition, consider the special case when implementing BBP is costless (i.e., $F = 0$), which is the case considered in the existing BBP literature. Recall that the perfect-information equilibrium is that the firm does not practice BBP (i.e., $s = \emptyset$), charges $p_1 = p_2 = \frac{1}{2}$, and gains a total profit of $\pi = \frac{1}{2}$. This equilibrium no longer holds when consumers cannot observe the firm's practice of BBP. Assume for contradiction that the perfect-information equilibrium still holds when consumers cannot observe the firm's choice s. Consider the following deviation: the firm deviates by choosing $\tilde{s} = \mathsf{BBP}$ in period 0 but still charges $\tilde{p}_1 = \frac{1}{2}$ in period 1. This guarantees that consumers do not observe any deviation in period 1 (they only observe $\tilde{p}_1 = p_1$). Therefore, consumers' beliefs are not affected: they hold the (incorrect) belief that the firm does not practice BBP. As such, all consumers with valuation $v_i \geq \frac{1}{2}$ will purchase the product, and the firm's period-1 profit is $\tilde{\pi}_1 = \frac{1}{4}$. In period 2, the firm can do better by price discriminating against consumers: it charges previous consumers a higher price $\tilde{p}_2^r = \frac{1}{2}$ and new consumers a lower price $\tilde{p}_2^n = \frac{1}{4}$. As a result, the firm's second-period profit is $\tilde{\pi}_2 = \frac{5}{16}$, and its total profit is $\tilde{\pi} = \tilde{\pi}_1 + \tilde{\pi}_2 = \frac{9}{16} > \pi = \frac{1}{2}$. Thus, the firm is better off deviating, and the assumed equilibrium does not exist.

Therefore, when consumers cannot observe the firm's choice s, the firm has an incentive to opportunistically practice BBP. By practicing BBP, the firm can price discriminate against consumers in period 2. Meanwhile, because consumers do not observe the deviation, the firm's deviation does not affect its period-1 profit. Thus, the ratchet effect in the perfect-information case does not apply. Therefore, when the cost of implementing BBP is low, choosing $s = \mathsf{BBP}$ is beneficial at the beginning of the second period: the firm guarantees a higher period-2 profit through the price discrimination effect without affecting its period-1 profit.

Following the same logic, given any price p_1, when the cost of implementing BBP is low (i.e., $F < \frac{7-2\sqrt{10}}{45}$), the value of price discrimination is above the cost of BBP in the second period. Therefore, the firm chooses to practice BBP to take advantage of the price discrimination effect. However, as all consumers are sophisticated, in equilibrium, they correctly anticipate that the firm has incentives to perform BBP in the second period. As a result, consumers account for the firm's opportunistic incentive, form their beliefs, and make purchase decisions accordingly. As Table 1 shows, practicing BBP turns out to be unprofitable compared with the firm's first-best solution under perfect information. Despite reduced profits with BBP, the firm has no means to escape from practicing BBP under imperfect information.

F **is medium: downward price distortion.** As long as the cost of implementing BBP is not low (i.e., $F \geq \frac{7-2\sqrt{10}}{45}$), the cost counters the benefit of price discrimination. As a result, the firm chooses not to invest in BBP. Interestingly, as Table 2 and Figure 2 show, when the cost of implementing BBP is medium (i.e., $\frac{7-2\sqrt{10}}{45} \leq F < \frac{1}{16}$), $p_1 < \frac{1}{2}$; that is, there is a downward distortion in the firm's period-1 price. The shaded area in Figure 2 illustrates the downward distortion in period-1 price from $\frac{1}{2}$. As Figure 2 shows, the distortion is most severe when $F = \frac{7-2\sqrt{10}}{45}$, decreases in F as F goes up, and finally vanishes when $F = \frac{1}{16}$. To understand the firm's first-period price distortion, we examine the relationship between the second-period value of price discrimination and the first-period price p_1.

In Figure 3, we plot the value of the price discrimination effect as a function of p_1, where

$$\text{The value of price discrimination} = \pi_2^{\text{BBP}} - \pi_2^{\emptyset} = \begin{cases} \frac{p_1^2}{4} & \text{if } p_1 < \frac{1}{2}, \\ p_1 - \frac{3p_1^2}{4} - \frac{1}{4} & \text{otherwise.} \end{cases}$$

In other words, the value of price discrimination is equal to the difference in second-period profits with and without BBP. As Figure 3 shows, the value of price discrimination is minimized at either low or high values of p_1 and is maximized when p_1 is moderate. The intuition for the result is as follows: in the extreme case when p_1 is 0 (1), all (no) consumers make an initial purchase in period 1, and the purchase history data contains virtually no information. In either case, the firm cannot price discriminate against consumers. At moderate p_1, however, there are both large segments of consumers who purchase and who do not purchase in period 1, and the purchase history data becomes more informative. As such, the firm can improve its profit substantially by price discriminating against the consumers.

The value of price discrimination is, however, not symmetric around $p_1 = \frac{1}{2}$, and it peaks at $p_1 = \frac{2}{3}$ (see Figure 3). This result is not obvious because the purchase history data is most informative at $p_1 = \frac{1}{2}$ in the "entropy" sense (Shannon and Weaver 1949). The intuition is as follows: to the firm, the information about high-valuation consumers is more useful than information about low-valuation consumers. In the extreme
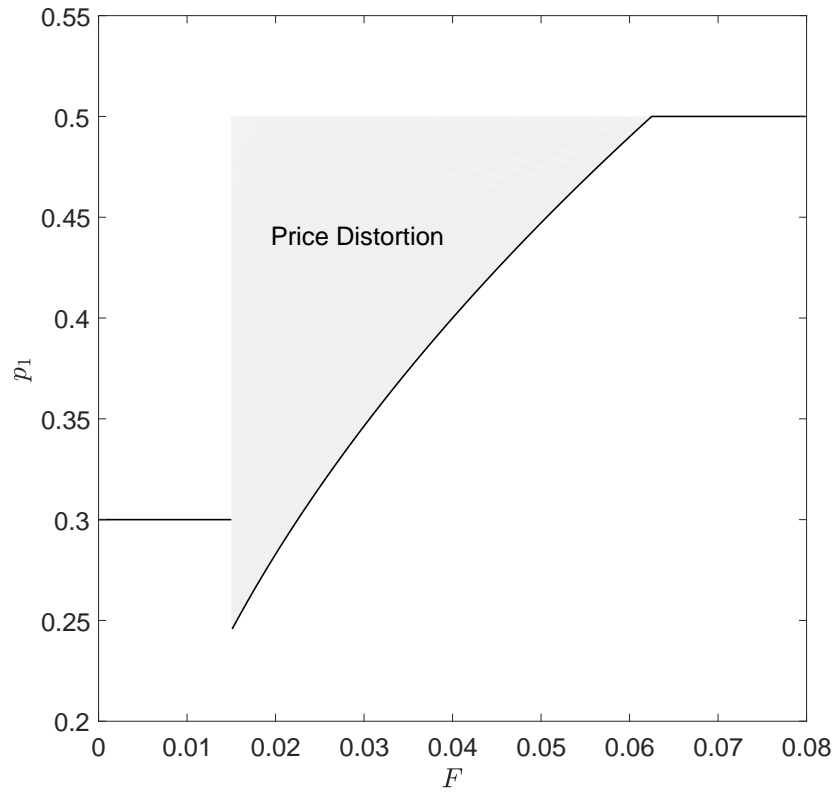
Figure 2   The firm's first-period price as a function of F
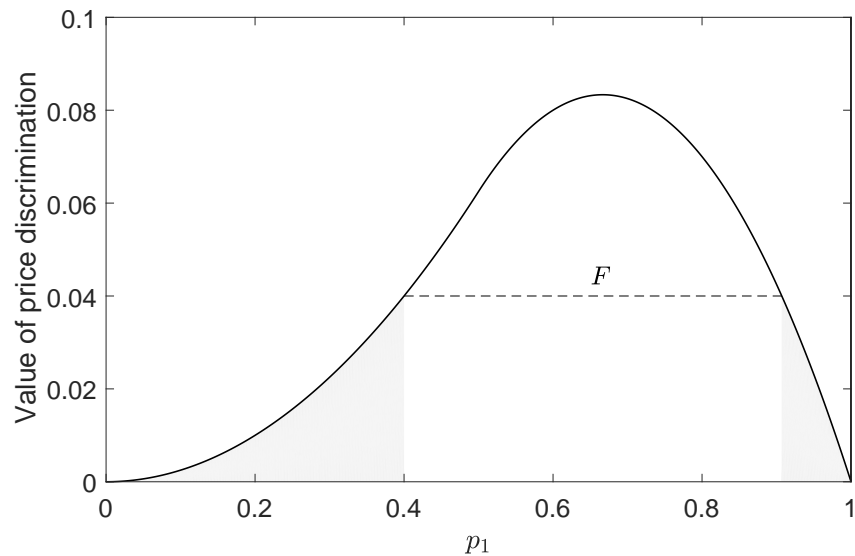

Figure 3   The value of price discrimination as a function of the first-period price

case, identifying a consumer of valuation $v_i = 0$ is not useful because that consumer will never be served in the market. When p_1 is high (but not too high), the firm gains more precise information about high-

valuation consumers and thus benefits more from that information. Thus, while purchase history data is most informative at $p_1 = \frac{1}{2}$, it is most valuable at $p_1 = \frac{2}{3}$.

In line with this logic, to avoid the BBP equilibrium, the firm must be willing to distort its first-period price to reduce the profitability of price discrimination (to make it lower than the cost of implementing BBP). As Figure 3 shows the firm can either distort its first-period price downward or distort it upward to make the price discrimination less valuable. Given the asymmetry in the value of price discrimination discussed previously, the firm prefers downward distortion to upward distortion, because the associated distortion is less severe when the firm distorts p_1 downward (for a detailed discussion, see the Appendix). In equilibrium, the firm distorts its price downward to $p_1 = 2\sqrt{F}$ to commit to a no-BBP equilibrium. Given that all consumers with valuation $v \in [2\sqrt{F}, 1]$ will make an initial purchase, the firm is indifferent about whether or not to practice BBP. Therefore, the price distortion is necessary to signal the firm's endogenous choice ($s = \emptyset$) to consumers.

F **is high: commitment device.** Finally, when the cost of implementing BBP is high (i.e., $F \geq \frac{1}{16}$), BBP becomes unprofitable, which enables the firm to commit to not price discriminating against consumers even without the need to distort its price. Therefore, a high cost serves as a commitment device for the firm. In equilibrium, the firm prices efficiently at $p_1 = \frac{1}{2}$ and achieves the first-best outcome. The resulting equilibrium is equivalent to the perfect-information benchmark.

Firm's Profit

When the cost of implementing BBP is low (i.e., $F < \frac{7-2\sqrt{10}}{45}$), the firm always practices BBP, and its total profit is given by

$$\pi = -F + (1 - \hat{v}) \cdot p_1 + \min\{1 - \hat{v}, 1 - p_2^r\} \cdot p_2^r + (\hat{v} - p_2^n)p_2^n, \tag{1}$$

where $\hat{v}$ is the indifferent consumer in period 1. Otherwise, the firm does not practice BBP, and its total profit is given by

$$\pi = (1 - p_1) \cdot p_1 + (1 - p_2) \cdot p_2. \tag{2}$$

Proposition 2 summarizes the results.

PROPOSITION 2. *Without transparency of BBP (i.e., under imperfect information), the firm's equilibrium profit π is as follows:*

(i) *When $F < \frac{7-2\sqrt{10}}{45}$, π decreases in F; when $\frac{7-2\sqrt{10}}{45} \leq F \leq \frac{1}{16}$, π increases in F.*

(ii) *π is minimized when $F = \frac{7-2\sqrt{10}}{45}$ and is maximized when $F \geq \frac{1}{16}$.*
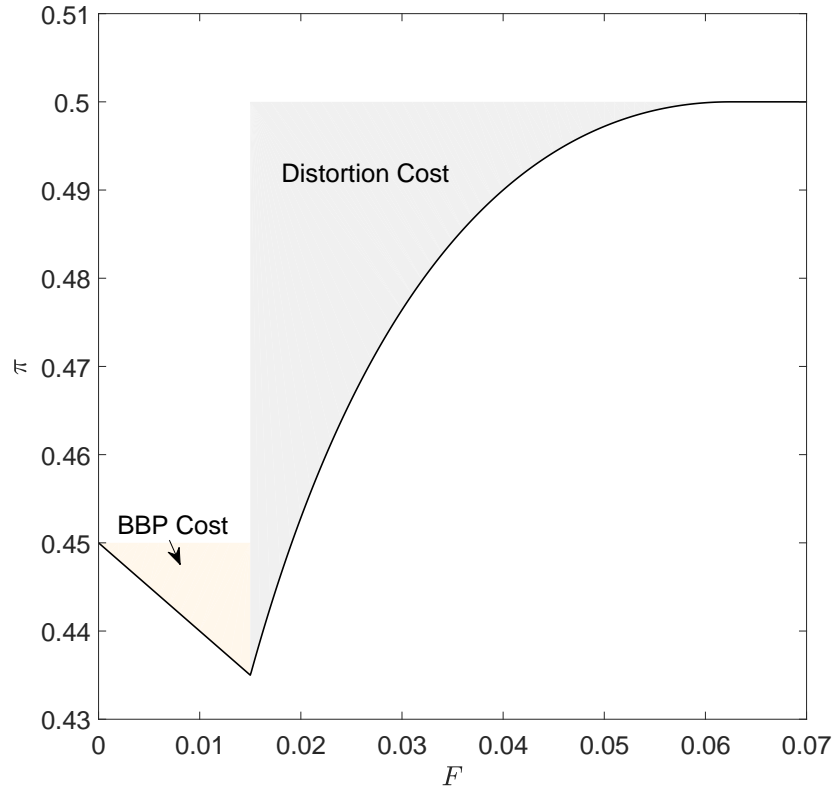
Figure 4   The firm's equilibrium profit as a function of F

Figure 4 illustrates the firm's equilibrium profit. Part (i) of Proposition 2 and Figure 4 suggest that the firm's profit is not monotone in F; that is, it decreases in F when F is low and increases in F otherwise. This result suggests that advances in information technologies do not always benefit firms. They can also reduce firm profit when F is moderate or high.

The rationale is as follows: when F is low, the firm cannot help opportunistically practicing BBP, as Proposition 1 suggests. As such, a higher F implies that the firm incurs a higher cost when implementing BBP, which affects the firm's profit negatively. This effect is represented by the first term on the right-hand side of Equation (1). By contrast, when F is moderate, the firm prefers the no-BBP equilibrium to the BBP equilibrium. However, to convince consumers that it does not practice BBP, the firm must distort its first-period price downward ($p_1 \leq \frac{1}{2}$), which makes the price discrimination less profitable. As F increases, practicing BBP becomes less profitable for the firm and the firm could signal its choice ($s = \emptyset$) to consumers more easily (i.e., with lesser distortion). This effect is represented by the first term on the right-hand side of Equation (2), which increases in F. As such, the firm benefits from an increase in F.

In line with this logic, when F is neither low nor high, both the BBP equilibrium and the no-BBP equilibrium are costly to achieve: to practice BBP, the firm must incur a considerable implementation cost; not

to practice BBP, the firm must distort its price severely to signal its choice to consumers. Part (ii) of Proposition 2 indicates that the firm's profit reaches its minimum of $\pi \approx 0.435$ when $F = \frac{7-2\sqrt{10}}{45}$, a 13% profit loss compared with the perfect-information benchmark. When F is high enough (i.e., $F \geq \frac{1}{16}$), BBP is too costly to implement, and the firm could signal its choice $s = \emptyset$ without distorting its first-period price. The firm's profit is maximized at $\pi = \frac{1}{2}$, which is equivalent to that in the perfect-information benchmark.

Consumer Surplus

Proposition 3 summarizes the result.

PROPOSITION 3. *Without transparency of BBP (i.e., under imperfect information), consumer surplus,*

CS, is given by

$$
CS = \begin{cases}
\frac{13}{40} & \text{if } F < \frac{7-2\sqrt{10}}{45}, \\[2mm]
\frac{5}{8} - 2\sqrt{F} + 2F & \text{if } \frac{7-2\sqrt{10}}{45} \leq F \leq \frac{1}{16}, \\[2mm]
\frac{1}{4} & \text{otherwise.}
\end{cases}
$$

where

(i) *CS is constant in F when $F < \frac{7-2\sqrt{10}}{45}$ or $F > \frac{1}{16}$ and decreases in F when $\frac{7-2\sqrt{10}}{45} \leq F \leq \frac{1}{16}$.*

(ii) *There is a discontinuous increase in CS at $F = \frac{7-2\sqrt{10}}{45}$.*

(iii) *CS is maximized when $F = \frac{7-2\sqrt{10}}{45}$ and is minimized when $F \geq \frac{1}{16}$.*

As Part (i) shows, consumer surplus is constant in F when F is either low or high. When F is low, the firm always charges the optimal BBP prices, whereas when F is high, the firm always charges the optimal no-BBP price (i.e., there is no price distortion). Therefore, prices and consumer surplus are constant in F. By contrast, when F is moderate, the firm prefers the no-BBP equilibrium but distorts its first-period price downward to signal its choice. The downward distortion in p_1 benefits consumers in two ways: First, a lower price increases the utility of consumers who purchase the product. Second, a lower price enables consumers who could not afford the product at a higher price to purchase it. As F increases, there is less price distortion (see Figure 2), and consumer surplus decreases.

Part (ii) of Proposition 3 suggests that there is a discontinuous increase in CS at $F = \frac{7-2\sqrt{10}}{45}$. This discontinuity showcases the switch from the BBP equilibrium to the no-BBP equilibrium (with price distortion) at $F = \frac{7-2\sqrt{10}}{45}$. Consumers benefit from the discontinuous decrease in the first-period price.

Part (iii) of Proposition 3 suggests that, compared with the firm's profit, which is maximized at high levels of the BBP implementation cost, consumer surplus reaches its maximum at a moderate BBP implementation
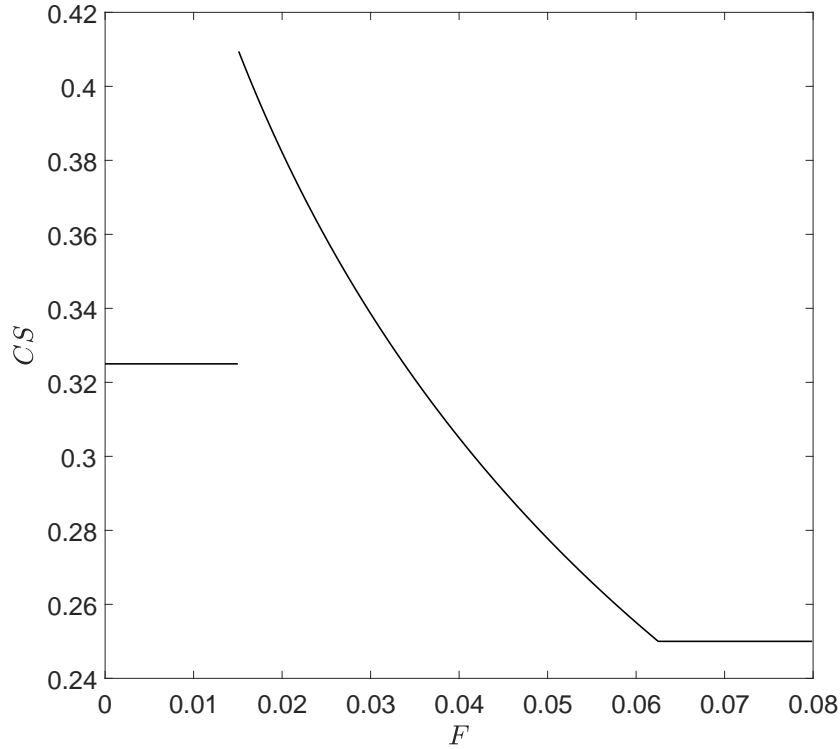
Figure 5   Equilibrium consumer surplus as a function of F

cost ($F = \frac{7-2\sqrt{10}}{45}$). At this cost, the firm decreases its first-period price significantly as a signal of its choice (i.e., $s = \emptyset$), and consumers benefit from such a low price. At $F = \frac{7-2\sqrt{10}}{45}$, consumer surplus reaches 0.410, a 64.0% improvement over the perfect-information benchmark. When $F \geq \frac{1}{16}$, the prices are too high in both periods, and consumer surplus suffers.

Social Welfare

Social welfare is the total value of firm profits and consumer surplus, whereas the cost of implementing BBP is a deadweight loss. Proposition 4 summarizes the result.

PROPOSITION 4. *Without transparency of BBP (i.e., under imperfect information), social welfare, SW,*

is given by

$$
SW = \begin{cases} \frac{31}{40} - F & \text{if } F < \frac{7-2\sqrt{10}}{45}, \\[2mm] \frac{7}{8} - 2F & \text{if } \frac{7-2\sqrt{10}}{45} \leq F \leq \frac{1}{16}, \\[2mm] \frac{3}{4} & \text{otherwise.} \end{cases}
$$

where

(i) *SW decreases in F when $F < \frac{7-2\sqrt{10}}{45}$ or $\frac{7-2\sqrt{10}}{45} \leq F < \frac{1}{16}$.*

(ii) *There is a discontinuous increase in SW at $F = \frac{7-2\sqrt{10}}{45}$.*

(iii) *SW is maximized when $F = \frac{7-2\sqrt{10}}{45}$ and is minimized when $F \geq \frac{1}{16}$.*
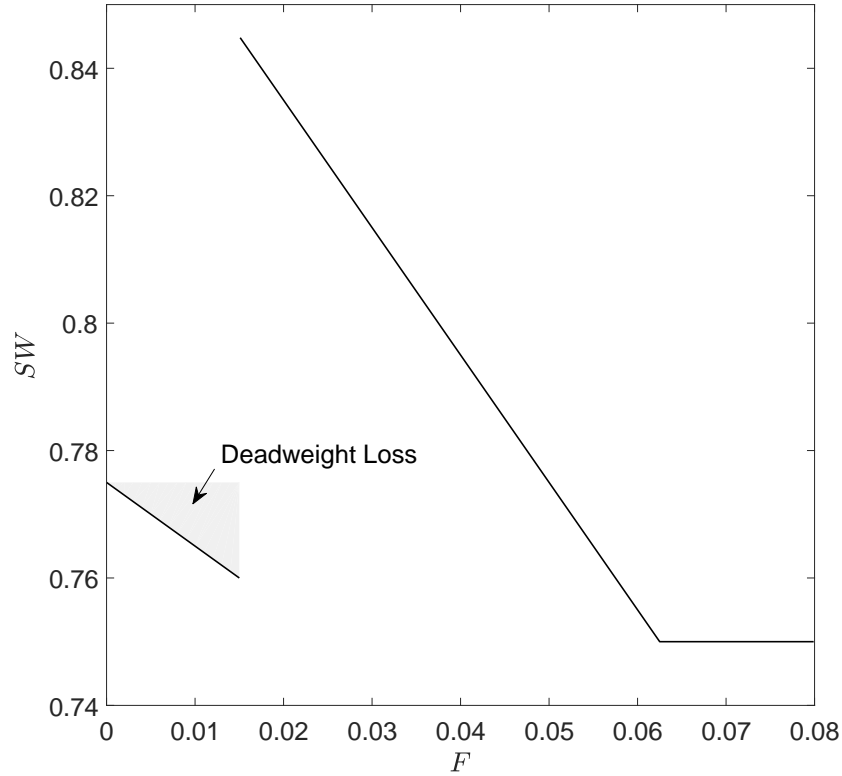

Figure 6   Equilibrium social welfare as a function of F

Part (i) indicates that social welfare decreases in F when F is either low (i.e., $F < \frac{7-2\sqrt{10}}{45}$) or moderate (i.e., $\frac{7-2\sqrt{10}}{45} \leq F < \frac{1}{16}$), but for very different reasons. When $F \leq \frac{7-2\sqrt{10}}{45}$, the firm practices BBP; social welfare decreases in F because a higher F exacerbates the deadweight loss in implementing BBP and makes the firm worse off (see Figure 6). When $\frac{7-2\sqrt{10}}{45} < F < \frac{1}{16}$, the firm does not practice BBP but distorts its first-period price downward to signal its choice. While this price distortion hurts the firm, it benefits consumers because the first-period price is lower, which enables some consumers who otherwise could not afford the product to purchase it. The expansion in demand increases social welfare. Overall, social welfare gains from this price distortion. As F grows, there is less price distortion, and p_1 increases, to the detriment of social welfare.

Part (ii) of Proposition 4 suggests that there is a discontinuous increase in SW at $F = \frac{7-2\sqrt{10}}{45}$. Again, this discontinuity showcases the switch from the BBP equilibrium to the no-BBP equilibrium at $F = \frac{7-2\sqrt{10}}{45}$.

This regime switch not only saves the BBP implementation cost but also leads to a discontinuous decrease in p_1, to the benefit of social welfare.

Consistent with Proposition 3, part (iii) of Proposition 4 shows that social welfare is maximized at $F = \frac{7-2\sqrt{10}}{45}$. At this cost, social welfare enjoys a 12.7% improvement over the perfect-information benchmark. When $F \geq \frac{1}{16}$, prices are too high in both periods, and social welfare suffers.[4]

Effects of Transparency of BBP

A direct comparison between regimes with and without transparency of BBP reveals the overall effects of BBP transparency on the firm, consumers, and social welfare.

PROPOSITION 5. *Transparency of BBP improves the firm's profit but decreases consumer surplus and social welfare.*

Transparency of BBP improves a firm's profit because it enables the firm to credibly commit not to practice BBP without having to distort its price downward. The transparency of its actions serves as a commitment device without having to resort to cutting prices and earning lower profits to signal commitment. As a result, the firm benefits from the data transparency regulation and achieves its first-best outcome. However, data transparency regulation, counter-intuitively, works to the detriment of consumer surplus. When F is low, consumer surplus is hurt because the first-period price is high when the firm forfeits price discrimination. When F is medium or high, consumer surplus is (weakly) hurt because the firm no longer distorts its price downward to signal its commitment to not practicing BBP, resulting in a higher first-period price and fewer consumers who can afford the product. For the same reason, although data transparency regulation helps the firm save the cost of implementing BBP, it reduces social welfare. Therefore, our key message is that from the standpoint of a consumer advocate or social planner, a BBP transparency regulation can be inefficient and must be applied with caution.[5]

PERSONALIZED ENHANCED SERVICE

A firm can go further than price discrimination when it bases marketing activity on consumers' past purchasing behavior. In this section, we extend the base model by considering the case in which the firm

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[4] If a third party collects the fee F for BBP implementation, our results still hold qualitatively. Specifically, when $F < \frac{7-2\sqrt{10}}{45}$, the firm always practices BBP, and social welfare is given by $SW = \frac{31}{40}$. When $F \geq \frac{7-2\sqrt{10}}{45}$, the firm does not practice BBP, and social welfare is the same as denoted in Proposition 4. Social welfare is still maximized when $F = \frac{7-2\sqrt{10}}{45}$. In the case when the third party endogenously sets the BBP fee F, our analysis suggests that the third party optimally sets the fee at $F = \frac{7-2\sqrt{10}}{45} - \varepsilon$ for some small $\varepsilon > 0$. Under such a fee, the firm's profit is minimized.

[5] Proposition 5 continues to hold when a third party collects the cost of implementing BBP.

could offer personalized enhanced service to previous buyers when it practices BBP. As Acquisti and Varian (2005) suggest, "such an enhanced service is based on information about the consumer's preferences. A consumer might frequent the same barber because that barber knows the consumer's preferences in haircuts. The barber, in turn, might charge a premium for his services because the consumer would have to incur costs in explaining to another barber exactly how his hair should be cut." In a similar vein, Pazgal and Soberman (2008) assume that, by practicing BBP, a firm can collect information from its consumers related to needs that are not addressed by the first-period offer and add a benefit to its second-period offer. With advances in information technologies, personalized enhanced services are becoming increasingly common, as firms can offer automated personalized enhanced services by taking consumer information directly from their databases. Note that personalized enhanced service can only be offered when there is a prior transaction between the consumer and the firm; otherwise, the firm has no means to assess the preferences or needs of an individual consumer.

Under perfect information, the firm cannot be worse off when it has the ability to offer consumers personalized enhanced service. If the firm practices BBP, it can attain higher profits in period 2 from offering personalized enhanced service to the previous consumers. If the firm does not practice BBP, it cannot offer personalized enhanced service and its profit will not be affected. In either case, being able to offer personalized enhanced service does not hurt the firm. However, as we show subsequently, this is not the case under imperfect information.

Formally, let $\Delta \geq 0$ denote the benefit of the personalized enhanced service that the firm offers to previous consumers (see Pazgal and Soberman 2008 for a similar assumption). The cost of offering such service is normalized to 0. The base model is thus a special case in which $\Delta = 0$. We focus on the interesting case when Δ is small enough so that the firm still prefers the no-BBP equilibrium under perfect information. We maintain the timing of the base model and continue to let F represent the cost of implementing BBP. We relegate the detailed analysis to the Appendix and present the equilibrium results in Proposition 6 and Table 3.

PROPOSITION 6. *Suppose that the firm is able to offer personalized enhanced service to previous consumers. Without transparency of BBP (i.e., under imperfect information), in equilibrium, the firm practices BBP when $F \leq F_1 = \frac{1}{45}(7 + 24\Delta + 12\Delta^2 - \sqrt{5}\sqrt{8 + 6\Delta + 3\Delta^2})$.*

As Proposition 6 indicates, consistent with the base model, in equilibrium the firm practices BBP when F is low and does not practice BBP otherwise. In addition, $\frac{\partial F_1}{\partial \Delta} > 0$; that is, personalized enhanced service leads to a wider range of situations for the firm to practice BBP. This is because the firm's ability to offer

Table 3   Equilibrium Strategies

	s	p_1	p_2
$F < F_1$	BBP	$\frac{3-2\Delta}{10}$	$p_2^r = \frac{3(1+\Delta)}{5}, p_2^n = \frac{3-2\Delta}{10}$
$F_1 \leq F \leq \frac{1+6\Delta+5\Delta^2}{16}$	$\emptyset$	$\sqrt{4F-2\Delta-\Delta^2}$	$\frac{1}{2}$
$\frac{1+6\Delta+5\Delta^2}{16} \leq F \leq \frac{1+8\Delta}{16}$	$\emptyset$	$\frac{2(1-\Delta)-\sqrt{(1+2\Delta)^2-12F}}{3}$	$\frac{1}{2}$
$\frac{1+8\Delta}{16} < F$	$\emptyset$	$\frac{1}{2}$	$\frac{1}{2}$

$$F_1 = \frac{1}{45}(7 + 24\Delta + 12\Delta^2 - \sqrt{5}\sqrt{8+6\Delta+3\Delta^2})$$

personalized enhanced service makes consumers' purchase history data more valuable. Here, consumers' purchase history data not only helps the firm price discriminate between previous and new consumers but also allows it to offer personalized enhanced service to previous consumers. As such, the firm finds BBP more profitable, and it is more willing to practice BBP at a moderate BBP implementation cost.

Next, we examine whether the firm benefits from its ability to offer personalized enhanced service. Specifically, we analyze the effect of Δ on the firm's profit and present the results in Proposition 7.

PROPOSITION 7. *The firm may be worse off if it can offer personalized enhanced service to consumers.*

As Figure 7 shows, $\Delta = 0$ corresponds to the case when the firm is unable to offer personalized enhanced service (or equivalently, consumers do not value personalized enhanced service at all). In terms of profit, the ability to offer personalized enhanced service positively affects the firm's profit at low levels of F but reduces profit beyond a critical value of F. Why is the firm worse off with the ability to offer personalized enhanced service at high levels of F?

Recall that the firm practices BBP when F is low and does not practice BBP when F is high. First, consider the case of a low F. The firm unambiguously benefits from its ability to offer personalized enhanced service. This is because, in equilibrium, the firm will practice BBP, and being able to offer personalized enhanced service allows the firm to charge higher prices to previous consumers, which raises the firm's second-period profit. Second, consider the case of a high F. The result is less obvious. As discussed previously, when F is high, the firm prefers the no-BBP equilibrium to the BBP equilibrium; however, to convince consumers that it does not practice BBP, the firm must distort its first-period price downward, making BBP less profitable. When the firm has the ability to offer personalized enhanced service, BBP becomes more profitable, and convincing consumers that it forfeits BBP becomes increasingly difficult. As a result, the firm must distort the first-period price more to signal its choice of forfeiting BBP, which affects its profit negatively. Mathematically, $\frac{\partial \pi}{\partial \Delta} = \frac{\partial \pi}{\partial p_1} \cdot \frac{\partial p_1}{\partial \Delta} \leq 0$ when $F \geq F_1$.
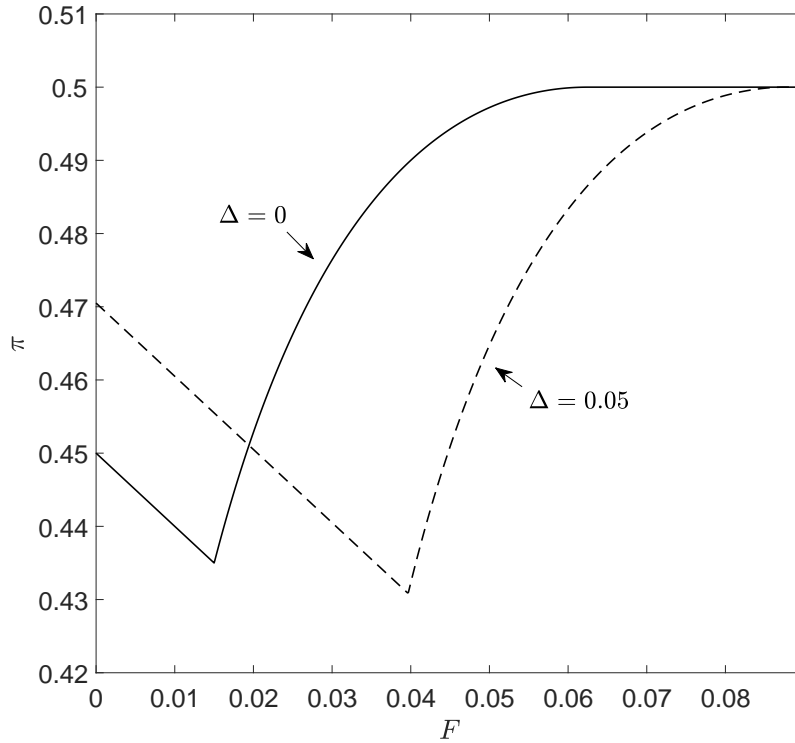
Figure 7 **The effect of personalized enhanced services on the firm's profit as a function of F**

DISCUSSION

Market Competition

Our model considers a monopolist firm. Would the main results hold under market competition? In compliance with the literature, we consider a duopoly setting with two firms, A and B, each selling a product to consumers in each of two periods, $t = 1, 2$. In period 0, the firms simultaneously decide whether to incur a cost F to invest in data collection and storage for BBP (i.e., $s_A, s_B \in \{\emptyset, \mathsf{BBP}\}$). A firm's BBP decision is not observed by the rival firm or the consumers. In period 1, the firms simultaneously choose their observed first-period prices p_{A1} and p_{B1}, and consumers make their initial purchase decisions. In period 2, contingent on their BBP decisions, the firms choose their second-period prices. That is, if firm $j \in \{A, B\}$ practices BBP, it offers two prices, p_{j2}^r and p_{j2}^n, to previous and new consumers, respectively. Otherwise, it offers a single price p_{j2} to all consumers.

The duopoly setup differs from the main model in two ways. First, a firm signals its BBP decision not only to consumers but also to the rival firm. Firm B's second-period price depends critically on its belief about firm A's BBP decision and vice versa. Second, under RI-refinement, there is no proper subgame in the reordered game in which the firms first choose their first-period prices and then make their BBP decisions. This is because firm B makes its BBP decision s_B without observing firm A's first-period price

p_{A1}. Likewise, firm A makes its BBP decision s_A without observing firm B's first period price p_{B1}. As such, subgame perfection does not pin down the out-of-equilibrium beliefs in the reordered game. These two substantially complicate the model, and we are not able to formally solve the duopoly model.

Despite this issue, our main insights should still hold in the duopoly setup: when the BBP implementation cost is low, the firms cannot help opportunistically practicing BBP. To see this, assume for contradiction that, in equilibrium, neither firm practices BBP and both charge p_{A1}^* and p_{B1}^* in period 1. Consider the following deviation: firm A secretly practices BBP but still charges p_{A1}^*. Because the deviation is not observed, the rival's and the consumers' beliefs about s_A should not change, and firm A's first-period profit will not be affected. However, in period 2, firm A enjoys an information advantage over its rival and makes a higher profit. Therefore, competition itself does not eliminate the opportunistic behavior of the firms. In line with this logic, when the cost of implementing BBP is high (but not too high), an equilibrium in which the firms do not practice BBP or distort their prices does not exist. Following the intuition from the base model, the firms may prefer not to practice BBP and distort their first-period prices to convince consumers of their choices. This reduces the value of consumers' purchase data, making BBP less profitable to practice.

Consumer Anonymity

Our main model assumes that consumers do not use anonymizing technologies to block data collection and avoid being recognized by firms. Conitzer, Taylor, and Wagman (2012) show that consumers' ability to adopt anonymizing technologies affects the firm's selling strategy. Here, we discuss how our results could continue to hold if we allow consumers to hide their identity. When consumers observe whether a firm practices BBP, given that a firm never practices BBP, there is no need for consumers to anonymize. When consumers do not observe whether the firm practices BBP, the problem is more complicated. If the cost of anonymity is negligible, all consumers anonymize, and the firm achieves its first-best solution (it does not practice BBP or distort its first-period price). If the cost of anonymity is prohibitive, no consumers anonymize, and the equilibrium results in our main model apply. Now consider the case of a moderate cost of anonymity. When the cost of implementing BBP (F) is low, a pure-strategy equilibrium in which the firm forfeits BBP does not exist. The intuition is as follows: assume for contradiction that such an equilibrium exists. Then, all consumers will be charged the same second-period price, and no first-period consumers will pay to anonymize. Given that the BBP implementation cost is low, the firm is better off practicing BBP to take advantage of the consumers' purchase history data, which contradicts the assumption. Thus, our main result that the firm cannot help practicing BBP when the BBP implementation cost is low still holds. In line with this logic, when the cost of implementing BBP is high (but not too high), the firm cannot achieve its first-best solution either. To avoid the BBP outcome, the firm can again distort its first-period

price to reduce the value of the price discrimination. As such, the firm commits to a no-BBP equilibrium, and no consumers anonymize.

Observability of F

Our model assumes that consumers cannot observe firm's decision to implement BBP but they know the cost F of implementing it. This assumption applies when resources such as industry or government reports can help consumers assess the cost of BBP implementation. For example, BCG published a report outlining the trend of hard drive storage costs and CPU processing capabilities which suggest the cost of implementing big data (Souza et al. 2013). Bantleman (2012) published general total costs to implement a big data analytics team of a certain scale. However, these reports may not always offer a precise estimate for some firms. Under these circumstances, consumers also need to consider the uncertainty of the cost while forming a belief on whether the firm practices BBP. Thus, it may be interesting to examine situations when consumers cannot observe the cost F but do have some prior belief or existing information on its distribution.

To model this information asymmetry between the firm and consumers, suppose that $F = 0$ with probability α and $F = F_0 > 0$ with probability $1 - \alpha$. The prior distribution of F is common knowledge in the market whereas the realization of F is not observed by consumers. Our basic model is thus the special case in which $\alpha = 0$. In this extension, consumers also must consider the prior distribution of F in addition to the price signal to form their beliefs. Clearly, if $F = 0$, the firm always has an incentive to practice BBP, which is the same as before. However, the firm can still choose a low p_1 to convince consumers that it will not practice BBP when $F = F_0$. In other words, as in the main model, a low p_1 signals a low likelihood of practicing BBP (i.e., α), which alleviates the ratchet effect and induces consumers to purchase early. Mathematically, let $\lambda = \Lambda(p_1)$ be the consumers' belief that $\Pr[s = \mathsf{BBP}|p_1]$, when F_0 is not too high, we come up with

$$
\lambda = \begin{cases}
\alpha & \text{when } p_1 \leq 2\sqrt{F_0}, \\
1 & \text{when } 2\sqrt{F_0} < p_1 < \frac{2+\sqrt{1-12F_0}}{6}, \\
\alpha + (1-\alpha)\frac{2(1+4F_0-2p_1+\sqrt{1-12F_0}p_1)}{1+4F_0} & \text{when } \frac{2+\sqrt{1-12F_0}}{6} \leq p_1 < \frac{2+\sqrt{1-12F_0}}{3}, \\
\alpha & \text{when } \frac{2+\sqrt{1-12F_0}}{3} \leq p_1,
\end{cases}
$$

and a price $p_1 \leq 2\sqrt{F_0}$ signals a low likelihood of implementing BBP. Therefore, the insights of our main model would continue to hold when F is not observed by consumers.

Dynamics and Endogenous Transparency

Our main model assumes that there is a single cohort of consumers that live for two periods. Now, consider a model with overlapping generations of consumers. In each period $t = 1, 2, \ldots$, one cohort of new consumers

arrive at the market, and they live for exactly two periods – periods t and $t + 1$ (for the same assumption, see Villas-Boas 1999). In each period, the firm decides whether or not to practice BBP. We assume that if the firm never practiced BBP before, it must incur a fixed cost F to practice BBP; otherwise, the firm does not need to incur any cost. This is because after the information infrastructures for BBP are built, the firm can use them in the future at minimal cost.

In the base model, when consumers arrive at the market, they observe nothing but the first-period price p_1 (which is the same to all consumers). Now, consumers arriving at period t have access to richer information; for example, a consumer can learn about the current period price charged to other consumers (which may differ from their own price when the firm practices BBP) and the past prices charged by the firm through word of mouth or price-tracking websites. Subsequently, they can infer whether the firm practices BBP in the current period or in previous periods. Unlike in the base model, now a period-t consumer's belief depends not only on the period-t price charged to him or her but also on the entire price history and the period-t price charged to other consumers. This naturally leads to a plethora of equilibria from various specifications of the out-of-equilibrium beliefs; yet existing refinement criteria are not sufficient to pin down the unique equilibrium.

While we are not able to analytically solve this model and refine the equilibria, we offer some insights into the new model. We argue that, when the firm is patient enough, there exists an equilibrium in which the firm commits to not practicing BBP without having to distort its prices, i.e., $p_t = \frac{1}{2}$ in each period t. Unlike in the base model, the equilibrium can be sustained even when $F = 0$. The consumer's belief is as follows: if the firm ever practiced BBP in the past, consumers believe that the firm will practice BBP in the future. Now consider the firm's incentive to practice BBP. If the firm deviates and practices BBP in period t, it would obtain an extra profit in period $t + 1$ through price discriminating against the period-t consumers. By contrast, all consumers arriving at or after period $t + 1$ believe that the firm will practice BBP (forever), and the ratchet effect emerges, which hurts the firm's profit. As such, the firm loses profits from all future consumers. When the firm is patient enough, this loss dominates and overshadows the profit gain from price discriminating against the period-t consumers, and the firm has no incentive to deviate. Interestingly, this equilibrium bears some similarities to the tacit collusion models: the firm and consumers coordinate to not practice BBP. If the firm defects and practices BBP, consumers do not trust the firm any longer and revert to the BBP equilibrium.

Alternatively, what will happen when a consumer can only observe the price that the firm offers to him or her? The intuitions will follow that of the base (two-period) model, but now the firm has a stronger incentive to implement BBP. The reason is that when BBP is implemented, the firm can use it forever for the purpose

of price discrimination, which will compensate for the implementation cost over the long run. When the firm is patient enough, it cannot help implementing BBP no matter how high the cost F is.

Another relevant question is, what happens when the firm has control over the transparency of its BBP decision? Our intuition is, whenever possible, the firm always prefers maximum transparency of its BBP decision. This arises because transparency helps the firm commit to not practicing BBP without incurring any additional costs. Therefore, transparency helps the firm obtain a higher profit.

CONCLUSION

As technology advances, firms can increasingly collect and use consumers' purchase history data for BBP. Firms often practice BBP without explicitly communicating such practices to consumers. Therefore, consumers are often unaware of firms' exploitation of their purchase history data for BBP. The widespread but unobserved practice of BBP has drawn growing public attention and debate about whether regulations should require firms to disclose their BBP practice to consumers. Extant research on BBP commonly assumes that consumers can directly observe whether firms practice BBP (i.e., they assume perfect information). In this article, we investigate a firm's decision to implement BBP when consumers do not directly observe this decision (i.e., with imperfect information). Furthermore, we compare the market equilibria in the perfect-information regime and the imperfect-information regime to evaluate the implications of BBP transparency on firms and consumers. Our results provide several insights and implications.

When consumers do not observe whether a firm practices BBP, how does the firm make BBP and pricing decisions? Our analysis shows that when the cost of implementing BBP is low, the firm cannot help practicing BBP. This occurs because, when consumers cannot observe the firm's choice, the firm has an incentive to opportunistically practice BBP, thereby benefiting from the price discrimination effect without affecting consumers' first-period purchase behavior. When the cost of implementing BBP is moderate, the firm does not practice BBP; however, it must distort its first-period price downward to convince consumers of its choice. When the cost of implementing BBP is high, the firm does not practice BBP or distort its price, as the benefit of BBP is offset by the high cost, which convinces consumers that the firm does not practice BBP. Therefore, when consumers do not observe the practice of BBP, a firm should make BBP and pricing decisions differently from situations when consumers observe BBP, and these decisions should depend on the cost of implementing BBP.

How does the cost of implementing BBP affect firm profit, consumer surplus, and social welfare? We find that the cost of implementing BBP does not exert a monotone impact on the firm's profit. The firm's profit decreases in the implementation cost but only up to a point, after which it increases with the implementation cost. This is due to the fact that the cost of implementation also serves as a commitment device to signal

the firm's choice. The cost of implementing BBP affects consumer surplus and social welfare in the reverse pattern of its impact on firm profit. Therefore, the declining cost of data storage and management for BBP could hurt firms but benefit consumers and society as a whole.

How does transparency of a firm's BBP practice affect firm profit, consumer surplus, and social welfare? Our comparison of the cases when consumers observe and do not observe BBP suggests that transparency of BBP improves firm profit at the cost of consumer surplus and social welfare. This is because mandatory disclosure of BBP serves as a commitment device that enables a firm to credibly commit to forfeiting BBP. As a result, the firm does not need to distort prices downward to costly signal to consumers about this decision. Therefore, transparency of BBP leads to higher prices, which benefits the firm but hurts consumers and society. This result implies that regulations that mandate that firms disclose the practice of BBP, designed to protect consumer privacy and welfare, could lead to unintended consequences.

How does a firm's ability to offer personalized enhanced service to consumers affect its profit? Our analysis shows that the ability to offer personalized enhanced service to consumers can either benefit or hurt the firm depending on the circumstances. When the BBP implementation cost is low, the firm practices BBP and gains from offering personalized enhanced service. However, when the BBP implementation cost is high, the firm's ability to offer personalized enhanced service reduces its profit. This is because, with this ability, it is increasingly costly for the firm to convince consumers that it does not practice BBP.

Future research could extend our study in several ways. First, the current model considers a monopolistic firm. Although we expect our main insights to hold in a competitive setting, it would be fruitful to glean additional insights from strategic competition. Second, the current model assumes that no consumers observe the firm's BBP decision. Research could further investigate the case when the firm's BBP decision is imperfectly observed. For example, Gavazza and Lizzeri (2009) consider the imperfect observation of political commitments. Third, similar to how consumers cannot observe the decision to practice BBP, they may also not know the true cost of implementation. While government and industry reports can aid in this, the cost of technology changes rapidly and exploring models that also assume the cost of implementation is not necessarily observable may yield interesting results. Fourth, our result implies that consumers are strategic and would interpret a low initial price as a signal of the no-BBP regime. Future research could assess the model's external validity by empirically examining how consumers interpret and respond to firms' initial pricing. Finally, we provided conjectural assessment of the dynamic model with overlapping generations of consumers and endogenous transparency decisions. It would be noteworthy to formally analyze the dynamic model and transparency decisions.

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Appendix

A.   Technical Details

Proof of Lemma 1: The case $s = \emptyset$ is straightforward, and we omit the proof. Now consider the case $s = \mathsf{BBP}$. Let v be the indifferent consumer from period 1. The indifference condition is specified as

$$v - p_1 + (v - p_2^r)^+ = v - p_2^n. \tag{3}$$

In period 2, the firm faces two segments of consumers: previous consumers (who bought the product) with valuation $v_i \geq v$ and new consumers (who did not buy the product) with $v_i < v$. Simple analysis shows that the optimal second-period prices are $p_2^r = \max\{v, \frac{1}{2}\}$ and $p_2^n = \frac{v}{2}$. Plugging them into Equation 3 yields $v = 2p_1$. Optimizing the firm's profit yields $p_1 = p_2^n = \frac{3}{10}$ and $p_2^r = \frac{3}{5}$. In equilibrium, the firm's profit from BBP is $\pi^{\mathsf{BBP}} = \frac{9}{20} - F$, where F is the BBP implementation cost.

Because $\pi^{\mathsf{BBP}} < \pi^\emptyset = \frac{1}{2}$ for all $F \geq 0$, we prove the lemma. Q.E.D.

Proof of Proposition 1: Under RI refinement, it suffices to consider the reordered game in which the firm first chooses the first-period price p_1 and then chooses s. We use subgame perfection to pin down consumers' belief about s given price p_1. In the analysis, we consider first pure-strategy equilibria and then mixed-strategy equilibria.

Case 1: $s = \emptyset$

Given p_1, suppose that there is a pure strategy equilibrium in which the firm always chooses $s = \emptyset$. In equilibrium, consumers hold the belief $\Lambda(p_1) = 0$. Under this belief, in period 1 all consumers with valuations $v_i \geq p_1$ make an initial purchase, and the firm's first-period profit is guaranteed to be $\pi_1 = p_1(1 - p_1)$ regardless of whether it practices BBP. Now consider how the firm's BBP decision affects its second-period profit.

- If $s = \emptyset$, the firm is not able to price discriminate against the consumers. The firm's optimal second-period price is $p_2 = \frac{1}{2}$, making a second-period profit of $\pi_2^\emptyset = \frac{1}{4}$.

- If $s = \mathsf{BBP}$, the firm is able to distinguish between two types of consumers: (1) previous consumers with valuations $v_i \in [p_1, 1]$ who made a purchase in period 1 and (2) new consumers with valuations $v_i \in [0, p_1)$ who did not buy. The firm charges a price p_2^r to the previous consumers and a price p_2^n to the new consumers. Simple calculation shows that the firm's optimal second-period prices are

$$p_2^r = \begin{cases} p_1 & \text{if } p_1 \geq \frac{1}{2}, \\ \\ \frac{1}{2} & \text{otherwise}, \end{cases}$$

and $p_2^n = \frac{p_1}{2}$. The firm's second-period profit is

$$\pi_2^{\mathsf{BBP}} = \begin{cases} (1-p_1)p_1 + \frac{p_1^2}{4} & \text{if } p_1 \geq \frac{1}{2}, \\ \\ \frac{1}{4} + \frac{p_1^2}{4} & \text{otherwise}. \end{cases}$$

In equilibrium, the firm has no incentive to deviate (i.e., it prefers not to practice BBP) iff $\pi^{\mathsf{BBP}} \leq \pi^\emptyset$, which translates

to $\pi_2^{\mathsf{BBP}} - F \leq \pi_2^\emptyset$. Solving the inequality, we obtain the existing conditions for the equilibrium: (1) $F \geq \frac{1}{12}$, or (2)

$p_1 \leq \underline{p_1}$, or (3) $p_1 \geq \frac{2+\sqrt{1-12F}}{3}$, where

$$\underline{p_1} = \begin{cases} 2\sqrt{F} & \text{if } F \leq \frac{1}{16}, \\ \\ \frac{2-\sqrt{1-12F}}{3} & \text{if } \frac{1}{16} \leq F \leq \frac{1}{12}. \end{cases}$$

Case 2: $s = \mathbf{BBP}$

Given p_1, suppose that there is a pure strategy equilibrium in which the firm always chooses $s = \mathsf{BBP}$. In equilibrium,

consumers hold the belief that $\Lambda(p_1) = 1$. Given this belief, let v denote the marginal consumer who is indifferent to

purchasing at $t = 1$. The indifference condition for the marginal consumer is

$$v - p_1 + (v - p_2^r)^+ = v - p_2^n,$$

where the left-hand side is the consumer's surplus if he or she buys in period 1 and the right-hand side is his or her

surplus if he or she does not buy in period 1. In equilibrium, $p_2^r = \max\{\frac{1}{2}, v\}$ and $p_2^n = \frac{v}{2}$, we obtain the following

condition:

$$v - p_1 = \frac{v}{2}.$$

Solving the indifference condition yields $v = 2p_1$, and the firm's first-period profit is

$$\pi_1 = \begin{cases} (1-2p_1)p_1 & \text{if } p_1 \leq \frac{1}{2}, \\ \\ 0 & \text{otherwise}, \end{cases}$$

regardless of whether it practices BBP. Note that when $p_1 > \frac{1}{2}$, no consumers buy at $t = 1$, and thus we say the indifferent consumer is located at $v = 1$.

Now consider the firm's second-period profit. Again, consider the following two cases.

- If $s = \emptyset$, the firm cannot price discriminate against the consumers. Its optimal strategy is $p_2 = \frac{1}{2}$, leading to a second-period profit of $\pi_2^{\emptyset} = \frac{1}{4}$.

- If $s = \mathsf{BBP}$, the firm can distinguish between two types of consumers: (1) previous consumers with valuations $v_i \in [v, 1]$ and (2) new consumers with valuations $v_i \in [0, v)$. The firm optimally charges p_2^r to previous consumers (when $p_1 > \frac{1}{2}$, there are no previous consumers) and p_2^n to new consumers, where

$$
p_2^r = \begin{cases} 2p_1 & \text{if } \frac{1}{4} \leq p_1 \leq \frac{1}{2}, \\[2mm] \frac{1}{2} & \text{otherwise,} \end{cases}
$$

and

$$
p_2^n = \begin{cases} \frac{1}{2} & \text{if } p_1 \geq \frac{1}{2}, \\[2mm] p_1 & \text{otherwise.} \end{cases}
$$

The firm's second-period profit is given by

$$
\pi_2^{\mathsf{BBP}} = \begin{cases} \frac{1}{4} & \text{if } p_1 \geq \frac{1}{2}, \\[2mm] 2p_1 - 3p_1^2 & \text{if } \frac{1}{4} \leq p_1 \leq \frac{1}{2}, \\[2mm] \frac{1}{4} + p_1^2 & \text{otherwise.} \end{cases}
$$

In equilibrium, the firm has no incentive to deviate (i.e., it prefers to practice BBP) iff $\pi^{\mathsf{BBP}} \geq \pi^{\emptyset}$, which translates to $\pi_2^{\mathsf{BBP}} - F \geq \pi_2^{\emptyset}$. Solving the inequality yields the existing conditions for the equilibrium: $F \leq \frac{1}{12}$ and

$$
\begin{cases} \sqrt{F} \leq p_1 \leq \frac{2+\sqrt{1-12F}}{6} & \text{if } F \leq \frac{1}{16}, \\[2mm] \frac{2-\sqrt{1-12F}}{6} \leq p_1 \leq \frac{2+\sqrt{1-12F}}{6} & \text{if } \frac{1}{16} \leq F \leq \frac{1}{12}. \end{cases}
$$

Case 3: Mixed-Strategy Equilibria

Next, consider the case when the firm randomizes its choice between BBP and $\emptyset$. Let $\lambda = \Lambda(p_1)$ be the consumers' beliefs that $\Pr[s = \mathsf{BBP}|p_1]$. Let v be the indifferent consumer from period 1. In period 2, if $s = \emptyset$, the firm charges all consumers $p_2 = \frac{1}{2}$. If $s = \mathsf{BBP}$, the firm charges new consumers $p_2^n = \frac{v}{2}$ and previous consumers $p_2^r = \max\{\frac{1}{2}, v\}$. Therefore, the indifference condition is given by

$$v - p_1 + \lambda(v - p_2^r)^+ + (1 - \lambda)(v - p_2)^+ = \lambda(v - p_2^n)^+ + (1 - \lambda)(v - p_2)^+,$$

which can be simplified as

$$v - p_1 = \lambda \cdot \frac{v}{2}.$$

In a mixed-strategy equilibrium, the firm must be indifferent about practicing BBP or not; that is $\pi^{\mathsf{BBP}} = \pi^\emptyset$, or equivalently, $\pi_2^{\mathsf{BBP}} - F = \pi_2^\emptyset$. This leads to

$$\begin{cases} F = \frac{v^2}{4} & \text{if } v \le \frac{1}{2}, \\ F = v - \frac{3v^2}{4} - \frac{1}{4} & \text{otherwise.} \end{cases}$$

Solving the equilibrium, we have three mixed-strategy equilibria:

(i) $v = 2\sqrt{F}, \lambda = 2 - \frac{p_1}{\sqrt{F}}$, where the equilibrium is sustained if $F \le \frac{1}{16}$ and $\sqrt{F} \le p_1 \le 2\sqrt{F}$;

(ii) $v = \frac{2 - \sqrt{1-12F}}{3}, \lambda = \frac{2(1+4F-2p_1-\sqrt{1-12F}p_1)}{1+4F}$, where the equilibrium is sustained if $\frac{1}{16} \le F \le \frac{1}{12}$ and $\frac{2-\sqrt{1-12F}}{6} \le p_1 \le \frac{2-\sqrt{1-12F}}{3}$;

(iii) $v = \frac{2 + \sqrt{1-12F}}{3}, \lambda = \frac{2(1+4F-2p_1+\sqrt{1-12F}p_1)}{1+4F}$, where the equilibrium is sustained if $F \le \frac{1}{12}$ and $\frac{2+\sqrt{1-12F}}{6} \le p_1 \le \frac{2+\sqrt{1-12F}}{3}$.

Equilibrium Refinement

Given this analysis, there may be multiple equilibria for some p_1 — for each $p_1 \in [0, 1]$, the number of equilibria ranges from 1 to 3. When there are multiple equilibria, we select the equilibrium with the smallest $\Lambda(p_1)$. In other words, we favor the no-BBP equilibrium over the BBP equilibrium. The rationales are as follows. First, this equilibrium selection criterion maximizes the firm's profit and allows us show the best possible outcome for the firm. Second, when forward induction applies, this equilibrium selection criterion picks the same equilibrium as what the forward induction criterion picks.

The Equilibrium

Given the subgame-perfect equilibrium in the reordered game, the firm chooses p_1 that will maximize its profit. Consider first the case $F \le \frac{7-2\sqrt{10}}{45}$. According to the analysis above, we summarize the equilibrium outcome depending on the value of p_1:

- If $p_1 \le 2\sqrt{F}$, the firm does not implement BBP. The firm's profit is $\pi = (1 - p_1)p_1 + \frac{1}{4}$.
- If $2\sqrt{F} \le p_1 \le \frac{2+\sqrt{1-12F}}{6}$, the firm always implements BBP. The firm's profit is $\pi = (1 - 2p_1)p_1 + \max\{2p_1 - 4p_1^2, \frac{1}{4}\} + p_1^2 - F$.

- If $\frac{2+\sqrt{1-12F}}{6} \le p_1 \le \frac{2+\sqrt{1-12F}}{3}$, the firm randomizes between implementing BBP and not. The firm's profit is $\pi = (1 - \frac{2+\sqrt{1-12F}}{3})\frac{2+\sqrt{1-12F}}{3} + \frac{1}{4}$.

- If $\frac{2+\sqrt{1-12F}}{3} \le p_1$, the firm does not implement BBP. The firm's profit is $\pi = (1 - p_1)p_1 + \frac{1}{4}$.

Note that given p_1, the equilibrium described above is not necessarily unique. As described above, when there are multiple equilibria, we choose the equilibrium with the lowest consumer belief of BBP, i.e., with the lowest λ. Such an equilibrium maximizes the firm's profit.

Comparing the above cases, we find that the firm maximizes its profit by choosing $p_1 = \frac{3}{10}$. The firm's payoff is $\pi = \frac{9}{20} - F$. The out-of-equilibrium belief is as follows:

$$\Lambda(p_1) = \begin{cases} 0 & \text{if } p_1 \le 2\sqrt{F}, \\[2mm] 1 & \text{if } 2\sqrt{F} < p_1 < \frac{2+\sqrt{1-12F}}{6}, \\[2mm] \frac{2(1+4F-2p_1+\sqrt{1-12F}p_1)}{1+4F} & \text{if } \frac{2+\sqrt{1-12F}}{6} \le p_1 < \frac{2+\sqrt{1-12F}}{3}, \\[2mm] 0 & \text{otherwise.} \end{cases}$$

Likewise, we can calculate the equilibrium for other values of F. In summary, when $F \le \frac{7-2\sqrt{10}}{45}$, the firm practices BBP and charges at $p_1 = \frac{3}{10}$, making a profit of $\pi = \frac{9}{20} - F$. When $\frac{7-2\sqrt{10}}{45} < F \le \frac{1}{16}$, the firm does not practice BBP and charges a price $p_1 = 2\sqrt{F}$, making a profit of $(1 - 2\sqrt{F})2\sqrt{F} + \frac{1}{4}$. At $F = \frac{7-2\sqrt{10}}{45}$, the firm is indifferent between practicing BBP and charging $p_1 = \frac{3}{10}$ and forfeiting BBP and charging $p_1 = 2\sqrt{F}$. Finally, when $F \ge \frac{1}{16}$, the firm achieves the first-best outcome: it does not practice BBP, prices at $p_1 = \frac{1}{2}$, and makes a profit of $\pi = \frac{1}{2}$. This leads to the equilibrium strategy summarized in Table 2. Q.E.D.

Proof of Proposition 2. The proposition follows immediately from Table 2. Q.E.D.

Proof of Proposition 3. First, when $F < \frac{7-2\sqrt{10}}{45}$, the firm practices BBP. In period 1, the indifferent consumer is located at $v = 2p_1 = \frac{3}{5}$, and all consumers with valuation $v_i \ge v$ buy the product. In period 2, all previous consumers purchase the product at p_2^r, and all new consumers with valuation $v_i \ge p_2^n$ buy the product at p_2^n. Consumer surplus is

$$CS = \int_v^1 (x - p_1)dx + \int_v^1 (x - p_2^r)dx + \int_{p_2^n}^v (x - p_2^n)dx = \frac{13}{40}.$$

Second, when $F \ge \frac{7-2\sqrt{10}}{45}$, the firm does not practice BBP. In period 1, all consumers with valuation $v_i \ge p_1$ buy the product, and in period 2, all consumers with valuation $v_i \ge p_2$ buy the product. Consumer surplus is

$$CS = \int_{p_1}^{1} (x - p_1) dx + \int_{p_2}^{1} (x - p_2) dx = \begin{cases} \frac{5}{8} - 2\sqrt{F} + 2F & \text{if } \frac{7 - 2\sqrt{10}}{45} \leq F \leq \frac{1}{16}, \\ \frac{1}{4} & \text{otherwise.} \end{cases}$$

The proof follows immediately. Q.E.D.

Proof of Proposition 4. First, when $F < \frac{7 - 2\sqrt{10}}{45}$, the firm practices BBP. In period 1, the indifferent consumer is located at $v = 2p_1 = \frac{3}{5}$, and all consumers with valuation $v_i \geq v$ buy the product. In period 2, all previous consumers purchase the product at p_2^r, and all new consumers with valuation $v_i \geq p_2^n$ buy the product at p_2^n. Social welfare is

$$SW = \int_{v}^{1} x dx + \int_{p_2^n}^{1} x dx - F = \frac{31}{40} - F,$$

where the last term on the right-hand side is the deadweight loss in implementing BBP.

Second, when $F \geq \frac{7 - 2\sqrt{10}}{45}$, the firm does not practice BBP. In period 1, all consumers with valuation $v_i \geq p_1$ buy the product, and in period 2, all consumers with valuation $v_i \geq p_2$ buy the product. Social welfare is

$$SW = \int_{p_1}^{1} x dx + \int_{p_2}^{1} x dx = \begin{cases} \frac{7}{8} - 2F & \text{if } \frac{7 - 2\sqrt{10}}{45} \leq F \leq \frac{1}{16}, \\ \frac{3}{4} & \text{otherwise.} \end{cases}$$

The proof follows immediately. Q.E.D.

Proof of Proposition 5. Under a data transparency regulation, the firm achieves the perfect information benchmark; that is, it does not practice BBP and charges $p_1 = p_2 = \frac{1}{2}$. In equilibrium, the firm's profit is $\pi = \frac{1}{4}$ and consumer surplus is

$$CS = \int_{\frac{1}{2}}^{1} \left(x - \frac{1}{2}\right) dx + \int_{\frac{1}{2}}^{1} \left(x - \frac{1}{2}\right) dx = \frac{1}{4}.$$

Comparing these results with the equilibrium outcome under imperfect information (see Table 2 and Proposition 3), the proposition follows immediately. Q.E.D.

Proof of Proposition 6: As with the base model, we consider the reordered game in which the firm first chooses p_1 and then chooses s.

Case 1: $s = \emptyset$

Given price p_1, we first consider the pure strategy equilibrium in which the firm always chooses $s = \emptyset$. In equilibrium, consumers believe that $\Lambda(p_1) = 0$. Given this belief, the firm's first-period profit is always $\pi_1 = p_1(1 - p_1)$ regardless

of whether it practices BBP. Now consider the firm's second-period profit:

- If $s = \emptyset$, the firm's optimal second-period price is $p_2 = \frac{1}{2}$, making a profit of $\pi_2^{\emptyset} = \frac{1}{4}$.

- If $s = \mathsf{BBP}$, the firm's optimal second-period prices are

$$
p_2^r = \begin{cases} p_1 + \Delta & \text{if } p_1 \geq \frac{1-\Delta}{2}, \\ \\ \frac{1+\Delta}{2} & \text{otherwise,} \end{cases}
$$

and $p_2^n = \frac{p_1}{2}$. The firm's second-period profit is

$$
\pi_2^{\mathsf{BBP}} = \begin{cases} (1 - p_1)(p_1 + \Delta) + \frac{p_1^2}{4} & \text{if } p_1 \geq \frac{1-\Delta}{2}, \\ \\ \frac{(1+\Delta)^2}{4} + \frac{p_1^2}{4} & \text{otherwise.} \end{cases}
$$

In equilibrium, the firm has no incentive to deviate (i.e., it chooses $s = \emptyset$) iff $\pi^{\mathsf{BBP}} \leq \pi^{\emptyset}$, which translates to $\pi_2^{\mathsf{BBP}} - F \leq \pi_2^{\emptyset}$. Solving the inequality, we know that the equilibrium is sustained if (1) $F \geq \frac{(1+2\Delta)^2}{12}$, or (2) $p_1 \leq \underline{p_1}$, or (3) $p_1 \geq \frac{2(1-\Delta)+\sqrt{(1+2\Delta)^2 - 12F}}{3}$, where

$$
\underline{p_1} = \begin{cases} \sqrt{4F - 2\Delta - \Delta^2} & \text{if } F \leq \frac{1 + 6\Delta + 5\Delta^2}{16}, \\ \\ \frac{2(1-\Delta) - \sqrt{(1+2\Delta)^2 - 12F}}{3} & \text{if } \frac{1 + 6\Delta + 5\Delta^2}{16} \leq F \leq \frac{(1+2\Delta)^2}{12}. \end{cases}
$$

Case 2: $s = \mathsf{BBP}$

Given consumers' beliefs, let v be the marginal consumer who is indifferent to purchasing at $t = 1$. The indifference condition can be written as

$$
v - p_1 = v - p_2^n = \frac{v}{2},
$$

Thus, the indifferent consumer is located at $v = 2p_1$, and the firm's first-period profit is $\pi_1 = (1 - 2p_1)p_1$ if $p_1 \leq \frac{1}{2}$ regardless of its true type. Otherwise, if $p_1 > \frac{1}{2}$, no consumer buys at $t = 1$.

Now consider the firm's second-period profit:

- If $s = \emptyset$, its best strategy is $p_2 = \frac{1}{2}$, leading to a profit of $\pi_2^{\emptyset} = \frac{1}{4}$.

- If $s = \mathsf{BBP}$, the firm optimally charges p_2^r to previous consumers (when $p_1 > \frac{1}{2}$, there are no previous consumers) and p_2^n to new consumers, where

$$
p_2^r = \begin{cases} 2p_1 + \Delta & \text{if } \frac{1-\Delta}{4} \leq p_1 \leq \frac{1}{2}, \\ \\ \frac{1+\Delta}{2} & \text{otherwise,} \end{cases}
$$

and

$$p_2^n = \begin{cases} \frac{1}{2} & \text{if } p_1 \geq \frac{1}{2}, \\ \\ p_1 & \text{otherwise.} \end{cases}$$

The firm's profit is given by

$$\pi_2^{\mathsf{BBP}} = \begin{cases} \frac{1}{4} & \text{if } p_1 \geq \frac{1}{2}, \\ \\ 2p_1 - 3p_1^2 + \Delta - 2\Delta p_1 & \text{if } \frac{1-\Delta}{4} \leq p_1 \leq \frac{1}{2}, \\ \\ \frac{(1+\Delta)^2}{4} + p_1^2 & \text{otherwise.} \end{cases}$$

In equilibrium, the firm has no incentive to deviate (i.e., it chooses $s = \mathsf{BBP}$) iff $\pi^{\mathsf{BBP}} \geq \pi^\emptyset$, which is translates to $\pi_2^{\mathsf{BBP}} - F \geq \pi_2^\emptyset$. Solving the inequality, we know that the equilibrium is sustained if $F \leq \frac{(1+2\Delta)^2}{12}$ and

$$\begin{cases} \frac{\sqrt{4F - 2\Delta - \Delta^2}}{2} \leq p_1 \leq \frac{2(1-\Delta) + \sqrt{(1+2\Delta)^2 - 12F}}{6} & \text{if } F \leq \frac{1+6\Delta+5\Delta^2}{16}, \\ \\ \frac{2(1-\Delta) - \sqrt{(1+2\Delta)^2 - 12F}}{6} \leq p_1 \leq \frac{2(1-\Delta) + \sqrt{(1+2\Delta)^2 - 12F}}{6} & \text{if } \frac{1+6\Delta+5\Delta^2}{16} \leq F \leq \frac{(1+2\Delta)^2}{12}. \end{cases}$$

Case 3: Mixed-Strategy Equilibria

Next, consider the case in which the firm randomizes its choice between BBP and $\emptyset$. Let $\lambda = \Lambda(p_1)$ be the consumers' beliefs that $\Pr[s = \mathsf{BBP}|p_1]$. Let v be the indifferent consumer from period 1. In period 2, if $s = \emptyset$, the firm charges all consumers $p_2 = \frac{1}{2}$. If $s = \mathsf{BBP}$, the firm charges new consumers $p_2^n = \frac{v}{2}$ and previous consumers $p_2^r = \max\{\frac{1+\Delta}{2}, v + \Delta\}$. Therefore, the following equation characterize the indifference condition:

$$v - p_1 + \lambda(v + \Delta - p_2^r)^+ + (1 - \lambda)(v - p_2)^+ = \lambda(v - p_2^n)^+ + (1 - \lambda)(v - p_2)^+,$$

which can be simplified to

$$v - p_1 = \lambda \cdot \frac{v}{2}.$$

In a mixed-strategy equilibrium, the firm must be indifferent about practicing BBP or not. This leads to

$$\begin{cases} F = \frac{1}{4}(v^2 + 2\Delta + \Delta^2) & \text{if } v \leq \frac{1-\Delta}{2}, \\ \\ F = \frac{1}{4}(1 - v)(3v - 1 + 4\Delta) & \text{otherwise.} \end{cases}$$

Solving the equilibrium, we have three mixed-strategy equilibria:

(i) $v = \sqrt{4F - 2\Delta - \Delta^2}, \lambda = 2 - \frac{2p_1}{\sqrt{4F - 2\Delta - \Delta^2}}$, where the equilibrium is sustained if $F \leq \frac{1+6\Delta+5\Delta^2}{16}$ and $\lambda \in [0, 1]$;

(ii) $v = \frac{2-2\Delta-\sqrt{(1+2\Delta)^2-12F}}{3}, \lambda = \frac{2(1+4F-4\Delta+2\Delta p_1-2p_1-\sqrt{(1+2\Delta)^2-12Fp_1})}{1-4\Delta+4F}$, where the equilibrium is sustained if

$\frac{1+6\Delta+5\Delta^2}{16} \leq F \leq \frac{(1+2\Delta)^2}{12}$ and $\lambda \in [0,1]$;

(iii) $v = \frac{2-2\Delta+\sqrt{(1+2\Delta)^2-12F}}{3}, \lambda = \frac{2(1+4F-4\Delta+2\Delta p_1-2p_1+\sqrt{(1+2\Delta)^2-12Fp_1})}{1-4\Delta+4F}$, where the equilibrium is sustained if

$F \leq \frac{(1+2\Delta)^2}{12}$ and $\lambda \in [0,1]$.

Following the proof of Proposition 1, when there are multiple equilibria for the subgame, we select the equilibrium with the smallest $\Lambda(p_1)$. In other words, we favor the no-BBP equilibrium over the BBP equilibrium. Given the subgame equilibrium, the firm chooses p_1 that maximizes its entire profit. Solving the firm's problem yields the equilibrium strategy which is summarized in Table 3. Q.E.D.

Details on Downward Price Distortion

We consider the case when the firm does not want to practice BBP and examine how the firm commits to a no-BBP equilibrium. The distortionless solution for the firm is to price at $p_1 = p_2 = \frac{1}{2}$. In each period, the firm makes a profit of $p_t(1-p_t) = \frac{1}{4}$ and its total profit is $\frac{1}{2}$. The question is, given price $p_1 = \frac{1}{2}$, do consumers really believe that the firm does not practice BBP?

To answer this question, we assume that is an equilibrium in which the firm does not practice BBP and charges the distortionless price $p_1 = \frac{1}{2}$. Consumers hold equilibrium beliefs that the firm does not practice BBP. Now consider the following deviation: the firm charges $p_1 = \frac{1}{2}$ but secretly practices BBP. Because consumers do not observe the deviation, their beliefs and the firm's period-1 profit are not affected, and consumers with valuations $v_i \in [\frac{1}{2}, 1]$ purchase the good in period 1. Under the deviation, the firm optimally charges $p_2^r = \frac{1}{2}$ and $p_2^n = \frac{1}{4}$ to previous and new consumers in period 2, respectively. The firm's period-2 profit under deviation is $\frac{5}{16}$ ($\frac{1}{2} \times \frac{1}{2}$ from previous consumers and $\frac{1}{4} \times \frac{1}{4}$ from new consumers). In this case, the value of price discrimination is $\frac{5}{16} - \frac{1}{4} = \frac{1}{16}$, where $\frac{1}{4}$ is the firm's period-2 profit if it does not practice BBP. The firm has no incentive to deviate iff $F \geq \frac{1}{16}$. In other words, when $F \geq \frac{1}{16}$, the firm can signal its choice $s = \mathsf{BBP}$ using a distortionless period-1 price $p_1 = \frac{1}{2}$.

Given this analysis, when $F < \frac{1}{16}$, the firm cannot credibly commit to a no-BBP equilibrium at the distortionless price $p_1 = \frac{1}{2}$. To convince consumers that it does not practice BBP, the firm must choose a price $p_1 \neq \frac{1}{2}$. Assume that there is an equilibrium in which the firm does not practice BBP and charges a price p_1. Consumers hold the belief that the firm does not practice BBP. Again, consider the following deviation: the firm charges the equilibrium price p_1 but secretly practices BBP. Because consumers do not observe the deviation, the firm's period-1 profit is not affected, and

consumers with valuations $v_i \in [p_1, 1]$ purchase the good in period 1. In period 2, there are two types of consumers: previous consumers with valuations $v_i \geq p_1$ and new consumers with valuations $v_i < p_1$. Simple calculation shows that the deviating firm's optimal period-2 prices are $p_2^r = \max\{p_1, \frac{1}{2}\}$ and $p_2^n = \frac{p_1}{2}$, yielding a total profit of

$$\pi_2^{\mathsf{BBP}} = \max\left\{p_1(1-p_1), \frac{1}{4}\right\} + \frac{p_1^2}{4},$$

where $\max\left\{p_1(1-p_1), \frac{1}{4}\right\}$ is its profit from previous consumers and $\frac{p_1^2}{4}$ is its profit from new consumers. If the firm does not deviate, its period-2 profit is $\pi_2^{\emptyset} = \frac{1}{4}$. Therefore,

$$\text{The value of price discrimination} = \pi_2^{\mathsf{BBP}} - \pi_2^{\emptyset} = \begin{cases} \frac{p_1^2}{4} & \text{if } p_1 < \frac{1}{2}, \\ p_1 - \frac{3p_1^2}{4} - \frac{1}{4} & \text{otherwise.} \end{cases}$$

To guarantee that the firm has no incentive to deviate, we must impose the following condition

$$\text{The value of price discrimination} = \pi_2^{\mathsf{BBP}} - \pi_2^{\emptyset} \leq F.$$

Mathematically, when $F \leq \frac{1}{16}$, the no-deviating condition translates to

$$p_1 \leq \underline{p_1} = 2\sqrt{F} \quad \text{or} \quad p_1 \geq \overline{p_1} = \frac{2 + \sqrt{1 - 12F}}{3}.$$

In other words, to convince consumers that it does not practice BBP, the firm either distorts its price downward to $p_1 \leq \underline{p_1}$ or distorts its price upward to $p_1 \geq \overline{p_1}$. In either case, the firm successfully signals to consumers that it does not practice BBP. Its profit is $\pi = p_1(1-p_1) + \frac{1}{4}$. Therefore, when the firm wants to signal its no-BBP choice, its problem is

$$\max_{p_1} \; p_1(1-p_1) + \frac{1}{4},$$

$$\text{s.t. } p_1 \leq \underline{p_1} \text{ or } p_1 \geq \overline{p_1}.$$

Simple calculation shows that the firm chooses $p_1 = \underline{p_1}$; that is, it underprices in period 1.

Proof of Proposition 7. The proof follows directly from Figure 7 and the text. Q.E.D.