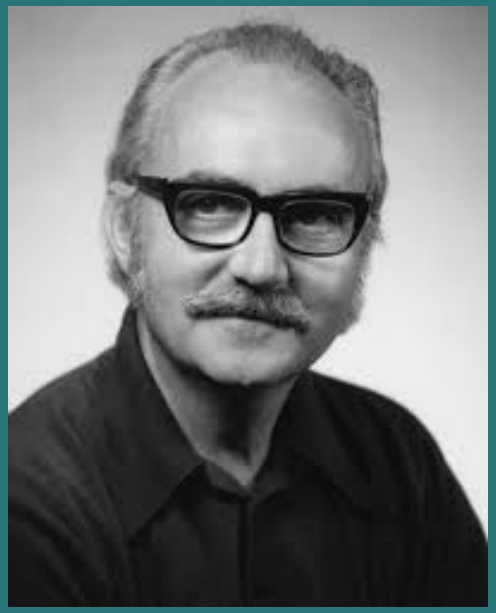
Discrete Choice

离散选择模型

All models are wrong, some are useful.

所有的模型都是错的，但有些是有用的。

--- George Box



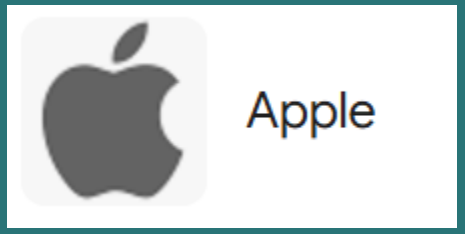
Question:

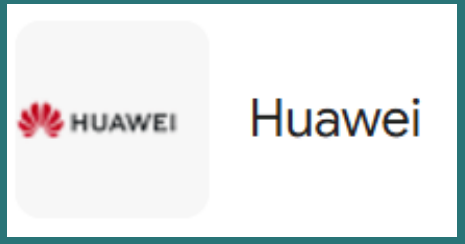
How do machines recognize hand-written digits?

机器是如何识别手写数字的?

What brand is my smartphone?

我的手机是什么品牌的?





What is the brand of the HKU president's car?

香港大学校长开什么牌子的车?

港聞 / 01偵查

港大校長座駕換車　張翔指定寶馬i7　205萬元豁免招標惹爭議

撰文：勞顯亮

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基于环保考虑，大学的校园设施管理部门建议以纯电动车代替燃油车。在比较了市面上多款电动车及混合动力型车的资料后，该部门与校长室挑选了以上两款，安排校长及两个部门的同事于同一天参与试车，并归纳了各人的反馈意见。校园设施管理部门同时比较了两款车的性能表现、车厢容量、外观、价格，亦考虑了大学的需要。

How do users choose among different banks?

用户是如何选择银行的?

Daniel McFadden's model became so popular, and he won the Nobel Prize in Economics in 2000 for "his development of theory and methods for analyzing discrete choice."

Daniel McFadden 建立了研究个人选择的模型。他的模型非常成功，为他赢得了2000年的诺贝尔经济学奖，颁奖词是“开创了研究离散选择模型的理论和方法。”

Modelling Consumer Choice

Human-beings always need to make choices, from your marriage choice to buying a bottle of milk.

While individuals can make choices in their own ways, as consumer analysts, we do want to understand how consumers make their choices.

消费者行为建模

人的一生离不开各种各样的选择，小到买哪个品牌的牛奶，大到如何选择自己的婚姻和职业。张雪峰?

每个人都用他们自己的方式进行各种选择。但是，我们还是希望尽可能的理解他们是如何进行选择的，并预测消费者的选择。

Imaging that you are a bank manager.

You want to understand how consumers choose between different credit card companies when applying for credit cards. In this way, you can understand which are really your potential clients, and you can target on these consumers better.

假设你是银行的经理...

你想知道大家是怎么去选择信用卡的，这样可以帮助你分析你的潜在客户，并改善你的产品来吸引更多的客户。

Your data is as follows...

For each consumer, you know his or her demographics (e.g., gender, age), occupation, income, geographic location, credit histories, etc. These are your independent variables.

You also know which credit card they applied to, e.g., Citibank, HSBC, BOC, American Express, ... or none of the above. This is your dependent variable.

You task: Building a model that predicts the dependent variable using your independent variables.

你的数据如下...

你知道每个消费者的人口统计学信息(例如年龄，性别，民族),职业，收入，地理位置，信用历史等等，你可以把他们作为你的自变量(解释变量)。

你也知道每个消费者申请了哪个银行的信用卡(建设银行，中国银行，工商银行，汇丰银行等)，而这是你的因变量(被解释变量)。

你的任务：建立你的统计学模型，通过自变量预测你的因变量。

What would you do?

你会怎么做?

Let us start with something simpler.

Now, you want to predict whether or not a consumer applies for your company's credit card. Here, the dependent variable Y_i is YES or NO. For simplicity, let $Y_i = 1$ for YES and $Y_i = 0$ for NO.

For each individual, the independent variables again include demographics, occupation, income, location, etc. We use X_i to denote the independent variables.

Our task: Predict Y_i using X_i.

我们先来点简单的...

我们考虑一个简单的问题，我们只分析消费者有没有申请建设银行的信用卡。这里，你的被解释变量 Y_i 的取值是 YES 或者 NO. 简单起见，我们用 $Y_i = 1$ 代表 YES，用 $Y_i = 0$ 代表 NO.

我们仍然知道每个消费者的人口统计学信息 (例如年龄，性别，民族), 职业，收入，地理位置，信用历史等等，并且用 X_i 表示这些被解释变量。

你的任务: 用 X_i 来预测 Y_i.

What should you do?

Our task: Predict Y_i using X_i, where $Y_i \in \{0, 1\}$.

Question: Can we use linear regression to analyze the relationship between Y_i and X_i, that is, we use the following linear model:

$$Y_i = \alpha + \beta X_i$$

我们该怎么做?

我们的任务：用 X_i 来预测 Y_i，其中 $Y_i \in \{0, 1\}$.

问题: 我们能不能用简单的线性回归来预测Y_i 和 X_i 的关系？这里，我们的回归方程是这样的:

$$Y_i = \alpha + \beta X_i$$

Issues with linear regression

Suppose that your regression result is:

$$Y_i = 0.4 + 0.1 \times Age_i + 0.2 \times Female_i$$

Suppose that a person's age is 25 and gender is male, you predict that his $Y_i = 0.65$, that is, the person is likely to buy from you.

线性回归的问题

假设你的回归结果是这样的：

$$Y_i = 0.4 + 0.1 \times Age_i + 0.2 \times Female_i$$

假设一个人的年龄是 25，性别是男性，根据回归公式我们的预测是 $Y_i = 0.65$, 换句话说，这个消费者有可能消费我们的产品。

Issues with linear regression

Suppose that your regression result is:

$$Y_i = 0.4 + 0.1 \times Age_i + 0.3 \times Female_i$$

Suppose that another person's age is 40 and gender is female, you predict that her $Y_i = 1.1$.

How would you interpret this result? Will she apply for your credit card 1.1 times? It does not make any sense!

线性回归的问题

假设你的回归结果是这样的：

$$Y_i = 0.4 + 0.1 \times Age_i + 0.2 \times Female_i$$

假设一个人的年龄是 40，性别是女性，根据回归公式我们的预测是 $Y_i = 1.1$。

你该如何解释这个结果？她一定会消费我们的产品？她会消费1.1次？

What should we do?

Instead of predicting the value of Y_i directly, we can predict the probability that Y_i is equal to 1, i.e., we want to predict $\Pr[Y_i = 1]$.

How to do that? We want to find out a function f such that

$$\Pr[Y_i = 1] \approx f(X_i)$$

Next, we will look for such a function f.

我们该怎么办?

与其直接预测 Y_i 的具体数值, 我们尝试预测 Y_i 等于 1 的概率, 即我们想预测 $\Pr[Y_i = 1]$.

我们需要找到一个这样的函数 f :

$$\Pr[Y_i = 1] \approx f(X_i)$$

现在，我们一起找找这样的函数 f.

What should we do?

How to do that? We want to find out a function f such that

$$\Pr[Y_i = 1] \approx f(X_i)$$

Here, we need to impose some restrictions on the function f:

1. $f(X) \geq 0$ for all X: probabilities are nonnegative.
2. $f(X) \leq 1$ for all X: probabilities are no more than 100%.
3. $f(X)$ is either increasing or decreasing with X.

我们该怎么办?

上一步我们说到我们希望找到这样的函数 f

$$\Pr[Y_i = 1] \approx f(X_i)$$

但并不是所有的函数 f 都可以。我们希望的函数需要满足一些条件:

1. $f(X) \geq 0$ for all X: 概率不能是负数。
2. $f(X) \leq 1$ for all X: 概率不能超过 100%.
3. $f(X)$ 对于 X 是递增或者递减函数.

What should we do?

$$\Pr[Y_i = 1] \approx f(X_i)$$

Here, we need to impose some restrictions on function f:

1. $f(X) \geq 0$ for all X: probabilities are nonnegative.
2. $f(X) \leq 1$ for all X: probabilities are no more than 100%.
3. $f(X)$ is either increasing or decreasing with X.

Can you propose such a function f? Any ideas?

我们该怎么办?

上一步我们说到我们希望找到这样的函数 f

$$\Pr[Y_i = 1] \approx f(X_i)$$

但并不是所有的函数 f 都可以。我们希望的函数需要满足一些条件:

1. $f(X) \geq 0$ for all X: 概率不能是负数。
2. $f(X) \leq 1$ for all X: 概率不能超过 100%.
3. $f(X)$ 对于 X 是递增或者递减函数.

你能找出这样的一个函数吗? 试试看!

$$f(X) = \frac{\exp(\alpha + \beta X)}{1 + \exp(\alpha + \beta X)}$$

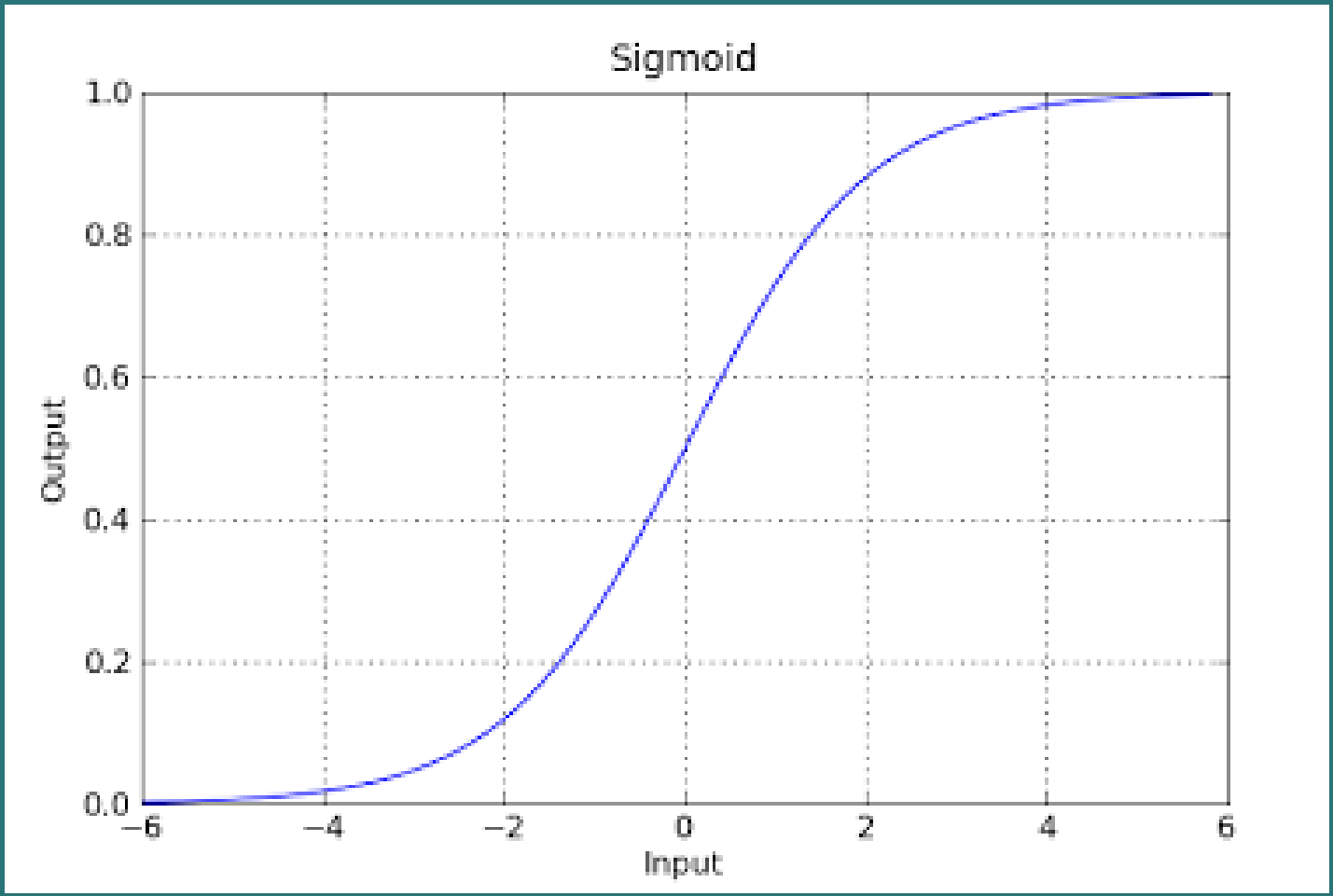
Note: $\exp(x) = e^x$ is the exponential function.

When $\beta > 0$, $f(X)$ increases with X; when $\beta < 0$, $f(X)$ decreases with X.

$$f(X) = \frac{\exp(\alpha + \beta X)}{1 + \exp(\alpha + \beta X)}$$

注释: $\exp(x) = e^x$ 是指数函数.

当$\beta > 0$ 时, $f(X)$ 是 X 的增函数; 当$\beta < 0$ 时, $f(X)$ 是 X 的减函数.



The logistic function 逻辑函数

Our task

We already know the values X_i and Y_i for each individual i. We would like to find the values of α and β to approximate the relationship between X_i and Y_i:

$$\Pr(Y_i = 1) \approx \frac{\exp(\alpha + \beta X_i)}{1 + \exp(\alpha + \beta X_i)}$$

This is done via maximum likelihood estimation.

我们的任务

现在，对于每一个消费者 i， 我们都知道对应的 X_i 和 Y_i 的数值. 我们希望进一步找到 α 和 β 的数值来近似 X_i 和 Y_i 之间的关系:

$$\Pr(Y_i = 1) \approx \frac{\exp(\alpha + \beta X_i)}{1 + \exp(\alpha + \beta X_i)}$$

这里，我们用一种最大似然估计的方法。

As an illustration, we first load the following dataset in R.

我们看看下面的数据：

```
mydata <- read.csv("https://ximarketing.github.io/data/loan.csv")
head(mydata, n = 20)
```

The data reads as follows:

具体的数据是这样的：

Age	Income	LoanAmount	CreditScore	MonthsEmployed	InterestRate	LoanTerm	Education	EmploymentType	MaritalStatus	HasMortgage	LoanPurpose	Default
56	85994	50587	520	80	15.23	36	Bachelors	Full-time	Divorced	Yes	Other	0
69	50432	124440	458	15	4.81	60	Masters	Full-time	Married	No	Other	0
46	84208	129188	451	26	21.17	24	Masters	Unemployed	Divorced	Yes	Auto	1
32	31713	44799	743	0	7.07	24	High School	Full-time	Married	No	Business	0
60	20437	9139	633	8	6.51	48	Bachelors	Unemployed	Divorced	No	Auto	0
25	90298	90448	720	18	22.72	24	High School	Unemployed	Single	Yes	Business	1

Age	Income	LoanAmount	CreditScore	MonthsEmployed	InterestRate	LoanTerm	Education	EmploymentType	MaritalStatus	HasMortgage	LoanPurpose	Default
56	85994	50587	520	80	15.23	36	Bachelors	Full-time	Divorced	Yes	Other	0
69	50432	124440	458	15	4.81	60	Masters	Full-time	Married	No	Other	0
46	84208	129188	451	26	21.17	24	Masters	Unemployed	Divorced	Yes	Auto	1
32	31713	44799	743	0	7.07	24	High School	Full-time	Married	No	Business	0
60	20437	9139	633	8	6.51	48	Bachelors	Unemployed	Divorced	No	Auto	0
25	90298	90448	720	18	22.72	24	High School	Unemployed	Single	Yes	Business	1

The data is about the loan default information, where the outcome is Default (1 = default, 0 = no default).

Education includes: high school, masters, bachelors, and PhD

Employment type includes: Full-time, part-time, unemployed, and self-employed

Marital status includes: single, married, and divorced

Loan purpose includes: auto, business, education, home, and other.

Age	Income	LoanAmount	CreditScore	MonthsEmployed	InterestRate	LoanTerm	Education	EmploymentType	MaritalStatus	HasMortgage	LoanPurpose	Default
56	85994	50587	520	80	15.23	36	Bachelors	Full-time	Divorced	Yes	Other	0
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60	20437	9139	633	8	6.51	48	Bachelors	Unemployed	Divorced	No	Auto	0
25	90298	90448	720	18	22.72	24	High School	Unemployed	Single	Yes	Business	1

这是某金融机构信用违约的数据。我们需要分析的变量是 Default (1 = default, 0 = no default).

Education includes: high school, masters, bachelors, and PhD

Employment type includes: Full-time, part-time, unemployed, and self-employed

Marital status includes: single, married, and divorced

Loan purpose includes: auto, business, education, home, and other.

```r
result <- glm(Default ~ Age + Income + LoanAmount +
CreditScore + MonthsEmployed + InterestRate + LoanTerm +
factor(Education) + factor(EmploymentType) +
factor(MaritalStatus) + factor(MaritalStatus) +
factor(HasMortgage) + factor(LoanPurpose), data = mydata,
family = "binomial")
summary(result)
```

Next, we build up a logistic regression model using default as the dependent variable, independent variables include all other variables.

```r
result <- glm(Default ~ Age + Income + LoanAmount +
CreditScore + MonthsEmployed + InterestRate + LoanTerm +
factor(Education) + factor(EmploymentType) +
factor(MaritalStatus) + factor(MaritalStatus) +
factor(HasMortgage) + factor(LoanPurpose), data = mydata,
family = "binomial")
summary(result)
```

接下来，我们建立一个逻辑回归模型。其中，我们的被解释变量为用户是否信用违约，而自变量包括其他所有变量。

	Estimate	Std. Error	z value	Pr(>|z|)	
(Intercept)	-3.683e-01	4.773e-02	-7.717	1.19e-14	***
Age	-3.920e-02	4.624e-04	-84.778	< 2e-16	***
Income	-8.753e-06	1.700e-07	-51.490	< 2e-16	***
LoanAmount	4.229e-06	9.314e-08	45.400	< 2e-16	***
CreditScore	-7.508e-04	4.093e-05	-18.344	< 2e-16	***
MonthsEmployed	-9.739e-03	1.912e-04	-50.929	< 2e-16	***
InterestRate	6.872e-02	1.017e-03	67.567	< 2e-16	***
LoanTerm	9.458e-05	3.820e-04	0.248	0.8045	
factor(Education)High School	7.828e-02	1.777e-02	4.404	1.06e-05	***
factor(Education)Masters	-1.301e-01	1.842e-02	-7.059	1.68e-12	***
factor(Education)PhD	-1.759e-01	1.853e-02	-9.490	< 2e-16	***
factor(EmploymentType)Part-time	2.816e-01	1.904e-02	14.791	< 2e-16	***
factor(EmploymentType)Self-employed	2.362e-01	1.921e-02	12.294	< 2e-16	***
factor(EmploymentType)Unemployed	4.416e-01	1.865e-02	23.673	< 2e-16	***
factor(MaritalStatus)Married	-2.267e-01	1.601e-02	-14.156	< 2e-16	***
factor(MaritalStatus)Single	-6.443e-02	1.557e-02	-4.139	3.49e-05	***
factor(HasMortgage)Yes	-1.560e-01	1.299e-02	-12.007	< 2e-16	***
factor(LoanPurpose)Business	4.425e-02	2.017e-02	2.193	0.0283	*
factor(LoanPurpose)Education	-1.687e-02	2.036e-02	-0.828	0.4075	
factor(LoanPurpose)Home	-1.935e-01	2.096e-02	-9.234	< 2e-16	***
factor(LoanPurpose)Other	-7.051e-03	2.038e-02	-0.346	0.7294	

How to interpret these results? 怎么解释这些结果?

We look at the estimates and the p-value (significance).

Age: older borrowers are less likely to default.

Income: Higher income implies lower likelihood of default.

Loan Amount: A higher amount implies higher chance to default.

Months Employed: Longer employment reduces the chance to default.

Interest Rate: High interest rate raises the chance to default.

Education: Higher degree reduces the chance to default.

Employment: Unemployed > part-time > self-employed > full-time

Marriage: Divorced > Single > Married

Mortgage: Mortgage increases the chance to default.

Loan Purpose: Business > Auto > Other > Education > Home

年龄：年长借款人违约的可能性较低。
收入：收入较高意味着违约的可能性较低。
贷款金额：较高的贷款金额意味着违约的几率较高。
就业月数：就业时间较长减少违约的机会。
利率：高利率提高违约的可能性。
教育：更高的学位减少违约的机会。
就业状况：失业 > 兼职 > 自雇 > 全职
婚姻状况：离婚 > 单身 > 结婚
按揭贷款：按揭贷款增加违约的可能性。
贷款目的：商业 > 汽车 > 其他 > 教育 > 住房

Probit regression

Probit 回归

Probit Regression

In logistic regression, we adopt the logistic function to estimate $\Pr[Y = 1 \mid X]$, which satisfies the properties that we listed. However, the logistic function is not the only function that satisfies those properties. Now, we introduce another function that can also make predictions about binary outcomes.

Probit 回归

在逻辑回归中，我们选择逻辑函数来描述关系$\Pr\left[Y=1 \middle| X\right]$，而这一函数满足我们之前提出的全部三个要求。但是，逻辑函数并不是唯一满足三个要求的函数。这里，我们再介绍一个新的函数，它同样满足这三个要求。

Probit Regression

Here, we use the cumulative distribution function of the standard normal distribution. Mathematically, suppose that $v \sim N(0, 1)$ is a standard normal random variable, then we can define the cumulative distribution function Φ as

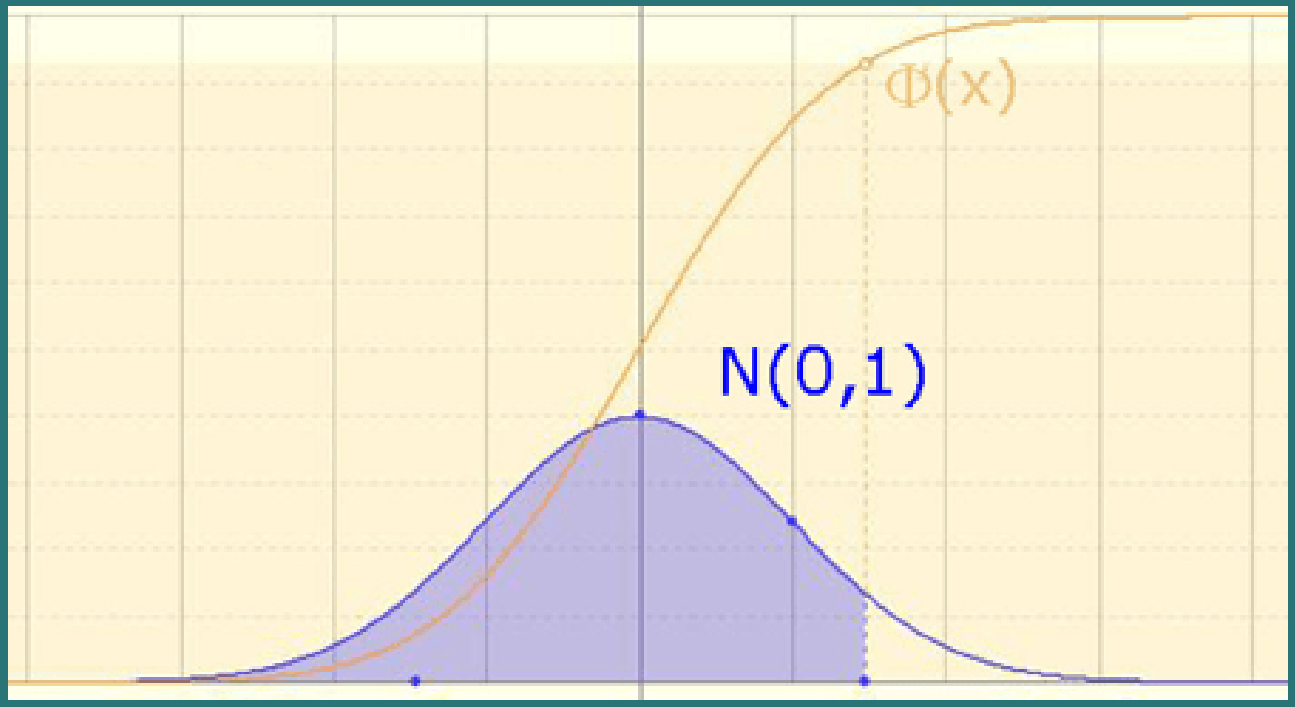
$$\Phi(z) = \Pr[v \leq z].$$

Probit 回归

这里，我们选择的函数是标准正态分布的累计分布函数。它的数学定义是这样的：假如随机变量 $v \sim N(0,1)$ 服从标准正态分布，那么它的累计分布函数 Φ 定义为

$$\Phi(z) = \Pr[v \leq z].$$

Probit Regression



Probit Regression

```
mydata <- read.csv("https://ximarketing.github.io/data/loan.csv")
head(mydata, n = 20)
result <- glm(Default ~ Age + Income + LoanAmount + CreditScore +
MonthsEmployed + InterestRate + LoanTerm + factor(Education) +
factor(EmploymentType) + factor(MaritalStatus) + factor(HasMortgage) +
factor(LoanPurpose), data = mydata, family = binomial(link="probit"))
summary(result)
```

Logistic vs. Probit

Question: Which one makes more sense?

你觉得哪个结果更合理?

	Default	
	logistic	*probit*
	(1)	(2)
Age	-0.039***	-0.021***
	(0.0005)	(0.0002)
Income	-0.00001***	-0.00000***
	(0.00000)	(0.00000)
LoanAmount	0.00000***	0.00000***
	(0.00000)	(0.00000)
CreditScore	-0.001***	-0.0004***
	(0.00004)	(0.00002)
MonthsEmployed	-0.010***	-0.005***
	(0.0002)	(0.0001)
InterestRate	0.069***	0.036***
	(0.001)	(0.001)
LoanTerm	0.0001	-0.00000
	(0.0004)	(0.0002)
factor(Education)High School	0.078***	0.041***
	(0.018)	(0.010)
factor(Education)Masters	-0.130***	-0.069***
	(0.018)	(0.010)
factor(Education)PhD	-0.176***	-0.092***
	(0.019)	(0.010)
factor(EmploymentType)Part-time	0.282***	0.148***
	(0.019)	(0.010)
factor(EmploymentType)Self-employed	0.236***	0.125***
	(0.019)	(0.010)
factor(EmploymentType)Unemployed	0.442***	0.232***
	(0.019)	(0.010)
factor(MaritalStatus)Married	-0.227***	-0.118***
	(0.016)	(0.009)
factor(MaritalStatus)Single	-0.064***	-0.032***
	(0.016)	(0.008)
factor(HasMortgage)Yes	-0.156***	-0.084***
	(0.013)	(0.007)
factor(LoanPurpose)Business	0.044**	0.024**
	(0.020)	(0.011)
factor(LoanPurpose)Education	-0.017	-0.008
	(0.020)	(0.011)
factor(LoanPurpose)Home	-0.194***	-0.102***
	(0.021)	(0.011)
factor(LoanPurpose)Other	-0.007	-0.004
	(0.020)	(0.011)

Logistic vs. Probit

They are similar models that yield similar (though not identical) inferences.

- Logistic regression is more popular in healthcare.
- Probit regression is more popular in political science.

But in most situations, it does not matter which method you choose to go with. Working with either will be fine.

Logistic vs. Probit

这两个模型会给你类似，但稍有不同的结果

- 逻辑回归常常被用在医疗分析上；
- Probit 回归在政治分析领域更加流行。

但在大多数情况下，你可以随便选择其中的一个模型，而具体的选择往往是一种习惯而不是基于任何理论。

The next question: What should we do when consumers have more than two choices?

当消费者有多于两个选择的时候，我们该怎么办?

More specifically, let us consider the following problem.

Each consumer i has his or her own information, which is measured by the independent variable X_i. The dependent variable is a choice made by the consumer, $Y_i \in \{A, B, \dots\}$.

现在，让我们考虑下面的问题：

我们知道每个消费者 i 的个人信息，而这些信息将成为我们的自变量 X_i. 因变量是每个消费者具体的选择，我们用 $Y_i \in \{A, B, \dots\}$ 来表示.

More specifically, let us consider the following problem.

Each consumer i has his or her own information, which is measured by the independent variable X_i. The dependent variable is a choice made by the consumer, $Y_i \in \{A, B, \dots\}$.

Idea: Instead of predicting Y_i directly, we predict the probability $\Pr[Y_i = A], \Pr[Y_i = B], \dots$

现在，让我们考虑下面的问题：

我们知道每个消费者 i 的个人信息，而这些信息将成为我们的自变量 X_i. 因变量是每个消费者具体的选择，我们用 $Y_i \in \{A, B, \dots\}$ 来表示.

思路：与其直接预测 Y_i，我们不如预测这些概率： $\Pr[Y_i = A], \Pr[Y_i = B], \dots$

Suppose that consumers have three choices, A, B, C.

Now, given X_i, we would like to come up with three functions $f_A(X_i)$, $f_B(X_i)$ and $f_C(X_i)$, such that

$$\Pr[Y_i = A] \approx f_A(X_i),$$
$$\Pr[Y_i = B] \approx f_B(X_i),$$
$$\Pr[Y_i = C] \approx f_C(X_i).$$

假设每个消费者有三个选择, 我们记为A, B, C.

现在，给定消费者的信息 X_i, 我们想找到三个函数 $f_A(X_i), f_B(X_i)$ 和 $f_C(X_i)$, 使得

$$\begin{aligned}
\Pr[Y_i = A] &\approx f_A(X_i), \\
\Pr[Y_i = B] &\approx f_B(X_i), \\
\Pr[Y_i = C] &\approx f_C(X_i).
\end{aligned}$$

As before, we place a few restrictions on these functions:

1. The probabilities must be nonnegative, i.e., $f_j(X_i) \geq 0$
2. Probabilities cannot exceed 1, i.e., $f_j(X_i) \leq 1$
3. Probabilities are monotone with X_i
4. Now, we have a new constraint: all the probabilities must add up to 100%, i.e.,

$$f_A(X_i) + f_B(X_i) + f_C(X_i) = 1.$$

Any ideas for the functions?

但是我们不能随便找三个函数。他们必须满足一定条件：

1. 所有的概率都不能是负数，即 $f_j(X_i) \geq 0$
2. 所有的概率都不能超过1，即 $f_j(X_i) \leq 1$
3. 所有的概率都是 X_i 的单调函数
4. 最后，我们还有一个额外的要求：消费者必须在这三个选项中选择一个，即

$$f_A(X_i) + f_B(X_i) + f_C(X_i) = 1.$$

你能找到这样的一组函数吗？

$$f_A(X_i) = \frac{\exp(\alpha_A + \beta_A X_i)}{\exp(\alpha_A + \beta_A X_i) + \exp(\alpha_B + \beta_B X_i) + \exp(\alpha_C + \beta_C X_i)}$$

$$f_B(X_i) = \frac{\exp(\alpha_B + \beta_B X_i)}{\exp(\alpha_A + \beta_A X_i) + \exp(\alpha_B + \beta_B X_i) + \exp(\alpha_C + \beta_C X_i)}$$

$$f_C(X_i) = \frac{\exp(\alpha_C + \beta_C X_i)}{\exp(\alpha_A + \beta_A X_i) + \exp(\alpha_B + \beta_B X_i) + \exp(\alpha_C + \beta_C X_i)}$$

They satisfy all the constraints! 它们符合左右的条件！

We need to estimate the values of α's and β's.

```r
install.packages("foreign")
install.packages("nnet")
install.packages("stargazer")

library(foreign)
library(nnet)
library(stargazer)
```

We install and load several packages for multinomial logit regression.

我们需要安装几个包来帮我们实现多项式逻辑回归模型(MNL模型)。

We first load the data from the Internet. 我们读取数据：

```
mydata <- read.csv("https://ximarketing.github.io/data/bankchoice.csv")
head(mydata, n = 20)
```

Here is the data... 数据是这样的：

	Choice	Age	Female	Income	Education	Job
1	CCB	62	0	7	2	Industry
2	CCB	34	1	4	5	Retired
3	CCB	68	0	5	2	Industry
4	CCB	60	0	3	2	Education
5	ICBC	18	0	6	2	Industry
6	CCB	18	0	4	3	Student
7	CCB	51	0	7	3	Education
8	CCB	25	1	3	2	Unemployed
9	BOC	42	1	4	4	Education
10	CCB	71	1	5	3	Service
11	BOC	23	0	4	5	Student
12	BOC	30	0	2	5	Retired

Here, we want to predict how individuals choose among the four major banks, ICBC, CCB, BOC and ABC.

The independent variables include the followings:

Age: Age of the consumer

Female: Whether or not the consumer is female (Female = 1)

Income: Income level from 1 (lowest) to 7 (highest)

Education: Education level from 1 (lowest) to 5 (highest)

Job: the job of the consumer, including Finance, Service, Education, Industry, Government, Unemployed, Retired

这里，我们想要预测个人在四大银行（工商银行、建设银行、中国银行和农业银行）之间的选择。

自变量包括以下内容：

- 年龄：消费者的年龄
- 性别：消费者是否为女性（女性 = 1）
- 收入：收入水平，从 1（最低）到 7（最高）
- 教育：教育水平，从 1（最低）到 5（最高）
- 职业：消费者的职业，包括金融、服务、教育、工业、政府、失业、退休

We use the multinom function to perform multinomial logit regression: 我们用multinom函数进行MNL模型分析

```r
result <- multinom(Choice ~ Age + Female +
                     Income + Education + factor(Job), data = mydata)
result
```

Oh, the results do not read nicely... 结果看起来不那么友好...

```
Coefficients:
     (Intercept)          Age      Female    Income Education factor(Job)Finance factor(Job)Government
BOC    0.5323798 -0.033020498 -0.08978578 0.6148213 1.2732409           4.298487             0.5149483
CCB    1.1092104 -0.018040309 -0.51117331 0.8868991 0.8061882           4.027794             0.8415596
ICBC  -0.5025889 -0.003286372 -0.37033922 0.5203970 0.2076094           5.033164             2.0033345
     factor(Job)Industry factor(Job)Retired factor(Job)Service factor(Job)Student factor(Job)Unemployed
BOC             2.439599         -2.5722926        -0.04396783         -1.0233035             0.2331643
CCB             2.154324         -3.1883673        -0.33249448         -1.4232623            -0.8674983
ICBC            4.733630         -0.6089712         1.28617434          0.7010564             1.2056360
```

No worries, let's try the stargazer function.

别担心，我们可以用stargazer函数来分析

```
stargazer(result, type="html",
out="result.html")
getwd()
```

Now, our results are nicely summarized in the table on the right-hand side:

What does it mean?

结果在我们的右表，它说明了什么？

	Dependent variable:		
	BOC	CCB	ICBC
	(1)	(2)	(3)
Age	-0.033***	-0.018***	-0.003
	(0.002)	(0.002)	(0.002)
Female	-0.090	-0.511***	-0.370***
	(0.077)	(0.076)	(0.079)
Income	0.615***	0.887***	0.520***
	(0.028)	(0.028)	(0.029)
Education	1.273***	0.806***	0.208***
	(0.038)	(0.038)	(0.039)
factor(Job)Finance	4.298***	4.028***	5.033***
	(0.053)	(0.051)	(0.076)
factor(Job)Government	0.515**	0.842***	2.003***
	(0.246)	(0.244)	(0.262)
factor(Job)Industry	2.440***	2.154***	4.734***
	(0.518)	(0.518)	(0.525)
factor(Job)Retired	-2.572***	-3.188***	-0.609***
	(0.145)	(0.143)	(0.167)
factor(Job)Service	-0.044	-0.332*	1.286***
	(0.188)	(0.187)	(0.209)
factor(Job)Student	-1.023***	-1.423***	0.701***
	(0.161)	(0.159)	(0.182)
factor(Job)Unemployed	0.233	-0.867***	1.206***
	(0.182)	(0.181)	(0.202)
Constant	0.532***	1.109***	-0.503**
	(0.200)	(0.198)	(0.221)
Akaike Inf. Crit.	81,191.300	81,191.300	81,191.300
Note:			*p<0.1; **p<0.05; ***p<0.01

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	BOC	CCB	ICBC
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Akaike Inf. Crit.	81,191.300	81,191.300	81,191.300
Note:		*p<0.1; **p<0.05; ***p<0.01	

Here, we take ABC as benchmark and compare other routes against it. Alternatively, you can view the parameters for ABC to be equal to 0.

Age: When consumer is older, she/he is less likely to choose BOC/CCB compared with ABC.

Female: Compared with CCB, females are more willing to choose ABC.

	Dependent variable:		
	BOC	CCB	ICBC
	(1)	(2)	(3)
Age	-0.033***	-0.018***	-0.003
	(0.002)	(0.002)	(0.002)
Female	-0.090	-0.511***	-0.370***
	(0.077)	(0.076)	(0.079)
Income	0.615***	0.887***	0.520***
	(0.028)	(0.028)	(0.029)
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	(0.182)	(0.181)	(0.202)
Constant	0.532***	1.109***	-0.503**
	(0.200)	(0.198)	(0.221)
Akaike Inf. Crit.	81,191.300	81,191.300	81,191.300
Note:			*p<0.1; **p<0.05; ***p<0.01

在这里，我们将农业银行（ABC）作为基准，并将其他银行与其进行比较。或者，您可以将ABC的参数视为等于0。

- 年龄：当消费者年龄较大时，与农业银行相比，她/他选择中国银行或建设银行的可能性较小。
- 性别：与建设银行相比，女性更倾向于选择农业银行。

The complete code is here:

```
library(foreign)
library(nnet)
library(stargazer)
mydata <- read.csv("https://ximarketing.github.io/data/bankchoice.csv")
head(mydata)
result <- multinom(Choice ~ Age + Female +
                     Income + Education + factor(Job),
                     data = mydata)
result
stargazer(result, type="html", out="result.html")
```

We first load the data from the Internet. 我们读取数据：

```
mydata <-
read.csv("https://ximarketing.github.io/data/multinomial_route_choice.csv")
head(mydata)
```

Here is the data... 数据是这样的：

	Choice	Flow	Distance	Seat_belt	Passengers	Age	Male	Income	Fuel_efficiency
1	Arterial	460	48	0	0	2	0	1	28
2	Rural	440	44	0	0	2	0	1	28
3	Freeway	130	61	0	0	2	0	1	28
4	Arterial	595	59	1	0	2	1	2	27
5	Rural	515	70	1	0	2	1	2	27
6	Freeway	340	87	1	0	2	1	2	27

Here, we want to predict how individuals choose the route when driving. The dependent variable is the chosen route, which can be arterial, rural, and freeway.

The independent variables include the followings:

Flow: A measure of traffic flow (how busy the traffic is).

Distance: The distance of the planned trip.

Seat_belt: whether the driver wears seat belt.

Passengers: Number of passengers carried.

Age: Age group of the driver.

Male: Whether the driver is male or not.

Income: Income level of the driver.

Fuel_efficiency: Fuel efficiency level of the vehicle.

这里，我们希望分析司机是如何选择道路的。每个司机的选项是我们的因变量，包括主干道(arterial)，乡间公路(rural)和高速路(freeway)三种选择。

自变量包括以下内容：

流量 Flow: 当前道路的繁忙情况.

里程 Distance: 需要驾驶路段的里程

安全带 Seat_belt: 司机有没有系安全带

乘客 Passengers: 车上有多少乘客.

年龄 Age: 司机的年龄.

男性 Male: 司机是否是男性.

收入 Income: 司机的收入水平.

燃油效率 Fuel_efficiency: 车辆的燃油效率.

We use the multinom function to perform multinomial logit regression: 我们用multinom函数进行MNL模型分析

```
result <- multinom(Choice ~ Flow + Distance +
                     Seat_belt + Passengers + Age + Male +
                     Income + Fuel_efficiency, data = mydata)
result
```

Oh, the results do not read nicely... 结果看起来不那么友好...

```
Coefficients:
        (Intercept)         Flow   Distance  Seat_belt Passengers         Age        Male
Freeway   13.673284 -0.049143703  0.1362782 -0.8924558  0.4775758  0.17728498  0.06331663
Rural      7.558223 -0.008436186 -0.0455514 -0.3451560  0.1436887 -0.06181751 -0.04244764
             Income Fuel_efficiency
Freeway -0.5430466     -0.06321059
Rural    0.1319585     -0.01778424
```

No worries, let's try the stargazer function.

别担心，我们可以用stargazer函数来分析

```
stargazer(result, type="html", out="result.html")
```

Now, our results are nicely summarized in the table on the right-hand side:

What does it mean?

结果在我们的右表，它说明了什么?

	Dependent variable:	
	Freeway	Rural
	(1)	(2)
Flow	-0.049***	-0.008***
	(0.006)	(0.001)
Distance	0.136***	-0.046***
	(0.031)	(0.014)
Seat_belt	-0.892	-0.345
	(0.663)	(0.319)
Passengers	0.478	0.144
	(0.454)	(0.275)
Age	0.177	-0.062
	(0.310)	(0.157)
Male	0.063	-0.042
	(0.638)	(0.302)
Income	-0.543	0.132
	(0.379)	(0.144)
Fuel_efficiency	-0.063	-0.018
	(0.068)	(0.038)
Constant	13.673***	7.558***
	(0.158)	(1.390)
Akaike Inf. Crit.	419.424	419.424
Note:	*p<0.1; **p<0.05; ***p<0.01	

	Dependent variable:	
	Freeway	Rural
	(1)	(2)
Flow	-0.049***	-0.008***
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	(0.031)	(0.014)
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Constant	13.673***	7.558***
	(0.158)	(1.390)
Akaike Inf. Crit.	419.424	419.424
Note:	*p<0.1; **p<0.05; ***p<0.01	

Here, we take arterial as the benchmark and compare other routes against it.
Alternatively, you can view the parameters for arterial to be equal to zero.

Flow: When there is a high flow, drivers are very less likely to choose freeway, and a bit less likely to choose rural compared with arterial.

Distance: When distance is long, drivers are more likely to choose freeway and less likely to choose rural route...

	Dependent variable:	
	Freeway	Rural
	(1)	(2)
Flow	-0.049***	-0.008***
	(0.006)	(0.001)
Distance	0.136***	-0.046***
	(0.031)	(0.014)
Seat_belt	-0.892	-0.345
	(0.663)	(0.319)
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Constant	13.673***	7.558***
	(0.158)	(1.390)
Akaike Inf. Crit.	419.424	419.424
Note:	*p<0.1; **p<0.05; ***p<0.01	

这里，我们将主干道为基准，将其它道路与主干道进行比较，并将对应的系数设为0.

Flow: 当车流量很大的时候，司机最不愿意驾驶高速路，最愿意驾驶主干道。

Distance: 当里程长的时候，司机最愿意驾驶高速路，最不愿意驾驶乡村公路。

The complete code is here:

```r
library(foreign)
library(nnet)
library(stargazer)
mydata <-
read.csv("https://ximarketing.github.io/data/multinomial_route_choice.csv"
)
head(mydata)
result <- multinom(Choice ~ Flow + Distance +
                     Seat_belt + Passengers + Age + Male +
                     Income + Fuel_efficiency, data = mydata)
result
stargazer(result, type="html", out="result.html")
```

Back to the Question:

回到之前的问题：

How do machines recognize hand-written digits?

机器是如何识别手写数字的？

Back to the Question:

How do machines recognize hand-written digits?

Absolutely, there are many sophisticated algorithms for handwriting recognition such as convolutional neural networks. But in the early stage, scientists just use the multinomial logit model to perform the task.

Input: Handwriting in pixels.

Output: $Y_i \in \{0, 1, \ldots, 9\}$

Absolutely, there are many sophisticated algorithms for handwriting recognition such as convolutional neural networks. But in the early stage, scientists just use the multinomial logit model to perform the task.

有很多好的算法可以帮助我们识别手写数字，比如卷积神经网络。但在早期阶段，计算机学家们使用的还是MNL模型。

Input 输入: Handwriting in pixels 一个一个像素的图像.

Output 输出: $Y_i \in \{0, 1, \ldots, 9\}$ 十个数字之一

Conditional Logit Model

条件Logit模型

In multinomial logit model, a person chooses among a few alternatives. The decision hinges on the decision maker's personal features, not the features of the alternatives. In our previous example, the route decision hinges on features such as distance, age, which are constant across all alternatives.

In conditional logit model, a person chooses among a few alternatives. The decision hinges on the alternatives' features, not the feature of the individuals.

在 multinomial logit model 模型, 一个人从几个选项中做出选择。 这个选择取决于这个人的个人特征而不是这些选项的特征，例如，选择基于这个人的年龄，性别等个人特征。

在 conditional logit model 模型, 一个人从几个选项中做出选择。这个选择取决于这个选项的特征而不是这个人的特征。例如，选择基于这个选项的价格，质量，颜色等。

Example:

Consumers choose among three computers, A, B, and C.

1. If the choices are based on consumers' age, gender, education etc, then we use the multinomial logit model.
2. If the choices are based on the price, quality of the computers, then we use the conditional logit model.

举例:

消费者从三个电脑品牌中选择一个, A, B, 和 C.

1. 如果选择是基于消费者的年龄，性别，职业等信息，那么我们选择的模型是 multinomial logit model.
2. 如果选择是基于每个电脑的价格，质量，服务等，那么我们选择的模型是 conditional logit model.

```r
install.packages("survival")
library(survival)
library(stargazer)
mydata = read.csv("https://ximarketing.github.io/data/conjoint.csv")
head(mydata)
```

id	interest	downpayment	rebate	speed	choice
1	3.75	40	0.15	0.5	0
1	4.00	25	0.15	1.0	0
1	3.75	25	0.00	1.0	1
2	3.50	20	0.10	0.5	1
2	3.75	25	0.30	1.5	0
2	3.75	20	0.30	1.0	0

id	interest	downpayment	rebate	speed	choice
1	3.75	40	0.15	0.5	0
1	4.00	25	0.15	1.0	0
1	3.75	25	0.00	1.0	1
2	3.50	20	0.10	0.5	1
2	3.75	25	0.30	1.5	0
2	3.75	20	0.30	1.0	0

Consumer 1 (id = 1) chooses between three offers:

Interest Rate	Down Payment	Rebate	Speed (Months)	Choice
3.75%	40%	0.15%	0.5	NO
4.00%	25%	0.15%	1.0	NO
3.75%	25%	0%	1.0	YES

id	interest	downpayment	rebate	speed	choice
1	3.75	40	0.15	0.5	0
1	4.00	25	0.15	1.0	0
1	3.75	25	0.00	1.0	1
2	3.50	20	0.10	0.5	1
2	3.75	25	0.30	1.5	0
2	3.75	20	0.30	1.0	0

用户1 (id = 1) 从下面三个按揭计划中做出选择:

按揭利率	首付	回赠	审批时间（月度）	选择
3.75%	40%	0.15%	0.5	NO
4.00%	25%	0.15%	1.0	NO
3.75%	25%	0%	1.0	YES

```
result<-clogit(choice ~ interest +  downpayment + rebate
               + speed + strata(id), data=mydata)
summary(result)
```

```
              coef exp(coef)  se(coef)      z Pr(>|z|)
interest  -1.185055  0.305729  0.097289 -12.181  < 2e-16 ***
downpayment -0.052922  0.948454  0.002336 -22.652  < 2e-16 ***
rebate     0.177522  1.194254  0.149303   1.189  0.23444
speed     -0.117274  0.889341  0.039587  -2.962  0.00305 **
```

```
stargazer(result, type="html", out="result.html")
```

	Dependent variable:
	choice
interest	-1.185***
	(0.097)
downpayment	-0.053***
	(0.002)
rebate	0.178
	(0.149)
speed	-0.117***
	(0.040)
Observations	18,000
R^2	0.039
Max. Possible R^2	0.519
Log Likelihood	-6,237.584
Wald Test	643.940*** (df = 4)
LR Test	708.180*** (df = 4)
Score (Logrank) Test	679.185*** (df = 4)
Note:	*p<0.1; **p<0.05; ***p<0.01

When interest rate increases, the user is less likely to choose the plan; when down payment increases, the user is less likely to choose the plan; when approval takes longer time, the user is less likely to choose the plan.

当利率增加，首付变高，或者审批时间变长的情况下，用户选择按揭的概率降低。

	coef	exp(coef)	se(coef)	z	Pr(>|z|)	
interest	-1.185055	0.305729	0.097289	-12.181	< 2e-16	***
downpayment	-0.052922	0.948454	0.002336	-22.652	< 2e-16	***
rebate	0.177522	1.194254	0.149303	1.189	0.23444	
speed	-0.117274	0.889341	0.039587	-2.962	0.00305	**

The coefficient for interest is -1.185 and the coefficient for speed is -0.1172. Because 0.1172/1.185=0.098, it suggests that a 1 month increase in approval time is equivalent to a 0.098% decrease in interest rate.

利率的系数是-1.185而审批时间系数是-0.1172. 因为0.1172/1.185=0.098，这说明审批时间增加一个月的代价相当于利率上涨0.098%元带来的代价。换句话说，一个月的审批时间对于消费者的价值是0.098%的利率。

The complete code is here:

```
library(survival)
library(stargazer)
mydata = read.csv("https://ximarketing.github.io/data/conjoint.csv")
head(mydata)
result<-clogit(choice ~ interest +  downpayment + rebate
               + speed + strata(id), data=mydata)
summary(result)
stargazer(result, type="html", out="result.html")
```

Predicting Market Share
预测市场占有率

Suppose that there are two plans available in the market:

Interest	Down Payment	Rebate	Speed
3.85%	30%	0.1%	1 month
4.25%	25%	0.25%	0.5 months

We can use our regression results to predict their market share, following the formula of conditional logit.

现在市场上有两种按揭产品，我们想预测它们的市场占有率

利率	首付	现金回赠	审批速度
3.85%	30%	0.1%	1 个月
4.25%	25%	0.25%	0.5 个月

```r
library(survival)
library(stargazer)
mydata = read.csv("https://ximarketing.github.io/data/conjoint.csv")
head(mydata)
result<-clogit(choice ~ interest +  downpayment +   rebate
               + speed + strata(id), data=mydata)
coef_interest <- coef(result)["interest"]
coef_downpayment <- coef(result)["downpayment"]
coef_rebate <- coef(result)["rebate"]
coef_speed <- coef(result)["speed"]

interest1 <- 3.85; downpayment1 <- 30; rebate1 <- 0.1; speed1 <- 1
interest2 <- 4.25; downpayment2 <- 25; rebate2 <- 0.25; speed2 <- 0.5

d1 <- exp(interest1 * coef_interest + downpayment1 * coef_downpayment +
            rebate1 * coef_rebate + speed1 * coef_speed)
d2 <- exp(interest2 * coef_interest + downpayment2 * coef_downpayment +
            rebate2 * coef_rebate + speed2 * coef_speed)

s1 <- d1/(d1+d2)
s2 <- d2/(d1+d2)
print(c(s1, s2))
```

讨论问题：

假设你要分析消费者如何选择银行(例如建设银行，工商银行，中国银行等)开立账户。但一个消费者可以选择多于一个银行开户。我们应该如何分析这个问题?

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答案很简单！用 Logistic 或者 Probit 回归

课后讨论问题：

如果我们知道消费者几个选项中最喜欢哪个选项，我们可以用离散选择模型来分析数据。但假如数据显示的是消费者最讨厌哪个选项而不是最喜欢哪个选项，我们还可以用离散选择模型分析这些数据吗？