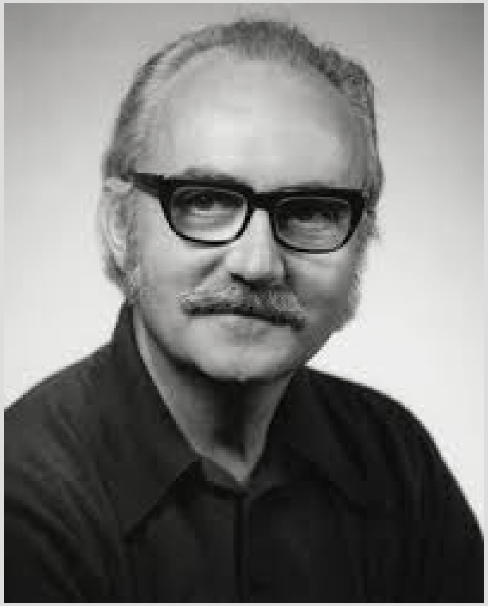
# Discrete Choice Models

*“ All models are wrong, some are useful.*

--- George Box



No model can capture the full complexity of the real world, yet a simplified model can still give good predictions or insight within a relevant range.

## How does AI recognize hand-written digits?

Daniel McFadden’s developed a model to understand our choices. His model became so popular, and he won the Nobel Prize in Economics in 2000 for “his development of theory and methods for analyzing discrete choice.”

# Modelling Consumer Choice

Human beings are constantly making choices—from deciding whom to marry to choosing a bottle of milk.

While individuals make decisions in their own ways, we want to understand how consumers make those choices.

Imaging that you are a bank manager...


You want to understand how consumers choose between different credit cards. In this way, you can understand who your potential clients are, and can target on these consumers better.

# Your data is as follows...

For each consumer, you know his or her demographics (e.g., gender, age), occupation, income, geographic location, credit histories, etc. These are your independent variables.

You also know which credit card they applied to, e.g., Citibank, HSBC, BOC, AMEX, ... or none of the above. This is your dependent variable.

You task: Building a model that predicts the dependent variable using your independent variables.

What would you do?

# Let us start with something simpler.

Now, you want to predict whether or not a consumer applies for your own company's credit card. Here, the dependent variable $Y_i$ is YES or NO. For simplicity, let $Y_i = 1$ for YES and $Y_i = 0$ for NO.

For each individual, the independent variables again include demographics, occupation, income, location, etc. We use $X_i$ to denote the independent variables.

Our task: Predict $Y_i$ using $X_i$.

# What should you do?

Our simplified task: Predict $Y_i$ using $X_i$, where $Y_i \in \{0, 1\}$.

Question: *Can we use linear regression to analyze the relationship between $Y_i$ and $X_i$, that is, we run the following linear model*:

$$Y_i = \alpha + \beta X_i$$

# What should we do?

Instead of predicting the value of $Y_i$ directly, we can predict the probability that $Y_i$ is equal to 1, i.e., we want to predict $\Pr[Y_i = 1]$.

How to do that? We want to find out a function $f$ such that

$$\Pr[Y_i = 1] \approx f(X_i)$$

Next, we look for such a function $f$.

# What should we do?

How to do that? We want to find out a function $f$ such that

$$\Pr[Y_i = 1] \approx f(X_i)$$

Here, we need to impose some restrictions on the function $f$:

1. $f(X) \geq 0$ for all $X$: probabilities are nonnegative.
2. $f(X) \leq 1$ for all $X$: probabilities are no more than 100%.
3. $f(X)$ is either increasing or decreasing with $X$.

# Our task

We already know the values $X_i$ and $Y_i$ for each individual $i$. We would like to find the values of $\alpha$ and $\beta$ to approximate the relationship between $X_i$ and $Y_i$:

$$\Pr(Y_i = 1) \approx \frac{\exp(\alpha + \beta X_i)}{1 + \exp(\alpha + \beta X_i)}$$

This is done via maximum likelihood estimation. Check here if you want to know more details.

As an illustration, we first load the following dataset in R.

```r
library(readr)
mydata <- read.csv("https://ximarketing.github.io/data/banking.csv")
head(mydata)
```

The data reads as follows:

| | age | job | previous | success |
|---|---|---|---|---|
| 1 | 44 | blue-collar | 0 | 0 |
| 2 | 53 | technician | 0 | 0 |
| 3 | 28 | management | 2 | 1 |
| 4 | 39 | services | 0 | 0 |
| 5 | 55 | retired | 1 | 1 |
| 6 | 30 | management | 0 | 0 |

| | age | job | previous | success |
|---|---|---|---|---|
| 1 | 44 | blue-collar | 0 | 0 |
| 2 | 53 | technician | 0 | 0 |
| 3 | 28 | management | 2 | 1 |
| 4 | 39 | services | 0 | 0 |
| 5 | 55 | retired | 1 | 1 |
| 6 | 30 | management | 0 | 0 |

The (real) data is about the outcome of a marketing campaign in a Portuguese bank that promotes a term deposit to their clients. Success denotes the final outcome of the campaign (1 = success, 0 = failure).

- Job includes admin, blue-collar, entrepreneur, housemaid, management, retired, self-employed, services, student, technician, unemployed, unknown.
- Previous denotes the number of previous interactions with the client. m deposit

```
# run a logistic regression, dv is success, iv is age, previous, data is mydata
model <- glm(success ~ age + previous, data = mydata, family = binomial)
```

```
result <- glm(success ~ age + factor(job) + previous, data
              = mydata, family = "binomial")
summary(result)
```

Next, we build up a logistic regression model using success to be the dependent variable, independent variables include age, job, and number of previous contacts.

Note that because "job" is not a number, we treat it as a fixed effect by enclosing it within a factor bracket.

```
Coefficients:
                          Estimate Std. Error z value Pr(>|z|)    
(Intercept)              -2.217305   0.074546 -29.744  < 2e-16 ***
age                       0.001890   0.001776   1.064 0.287149    
factor(job)blue-collar   -0.625683   0.051213 -12.217  < 2e-16 ***
factor(job)entrepreneur  -0.416913   0.099843  -4.176 2.97e-05 ***
factor(job)housemaid     -0.257722   0.109806  -2.347 0.018922 *  
factor(job)management    -0.174238   0.067648  -2.576 0.010005 *  
factor(job)retired        0.667628   0.078456   8.510  < 2e-16 ***
factor(job)self-employed -0.188631   0.093062  -2.027 0.042670 *  
factor(job)services      -0.487867   0.066121  -7.378 1.60e-13 ***
factor(job)student        0.879372   0.086922  10.117  < 2e-16 ***
factor(job)technician    -0.168579   0.050048  -3.368 0.000756 ***
factor(job)unemployed     0.093801   0.097300   0.964 0.335027    
factor(job)unknown       -0.150583   0.181433  -0.830 0.406558    
previous                  0.879022   0.024831  35.401  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

How to interpret these results?

We look at the estimates and the p-value (significance).

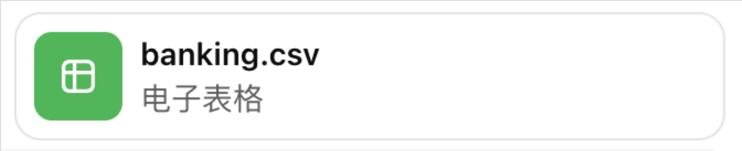
Age is not significant; it means whether a client accepts your promotion has little to do with his or her age.

Previous is significant and positive, meaning that getting a deal is easier when you have more previous interactions with the client.

Lastly, which types of jobs are more likely to accept your promotion? Retired and student. On the other hand, blue-collar, services, and entrepreneurs are unlikely to be convinced.

Using GPT 5:

Run a logistic regression. The dependent variable is success. The independent variables are age, previous, and job. Show the results.



## ⭐ Key Findings

### 1. Significant predictors

- **previous** (number of previous contacts)
  - **$\beta = 0.8790$, $p < 0.001$**
  - Strongest predictor: more previous contacts → higher probability of success.
- **Job categories with *higher* probability of success (vs. baseline job category):**
  - **student**, $\beta = 0.8794$, $p < 0.001$
  - **retired**, $\beta = 0.6676$, $p < 0.001$
- **Job categories with *lower* probability of success:**
  - blue-collar, services, technician, self-employed, housemaid, etc.
  - All statistically significant except unemployed and unknown.

### 2. Not significant

- **age** ($\beta = 0.0019$, $p = 0.287$)

  Age does **not** significantly predict success here.
- Some job categories (unemployed, unknown) are also not significant.

# Probit regression

# Probit Regression

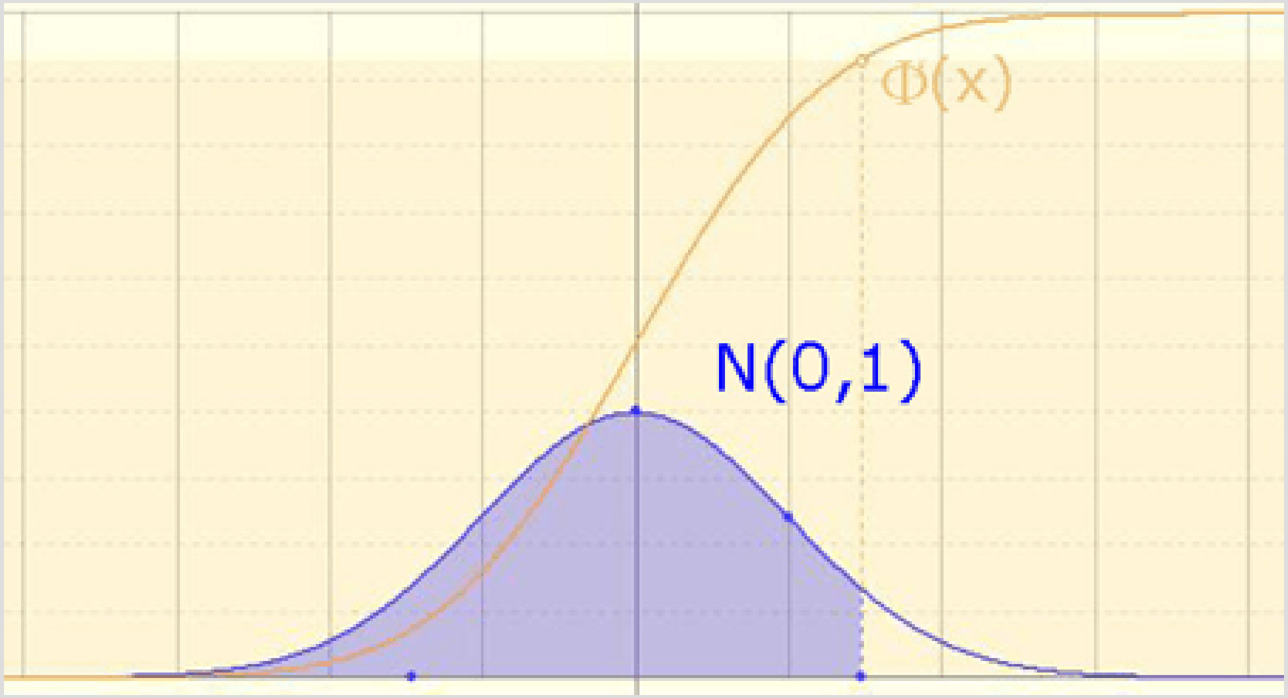
In logistic regression, we adopt the logistic function to estimate $\Pr\left[Y = 1 \,\middle|\, X\right]$, which satisfies the properties that we listed. However, the logistic function is not the only function that satisfies those properties. Now, we introduce another function that can also make predictions about binary outcomes.

# Probit Regression

Here, we use the cumulative distribution function of the standard normal distribution. Mathematically, suppose that $v \sim N(0,1)$ is a standard normal random variable, then we can define the cumulative distribution function $\Phi$ as

$$\Phi(z) = \Pr[v \leq z].$$

# Probit Regression

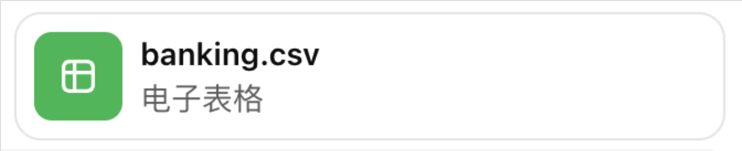


# Probit Regression

```r
library(readr)
mydata <- read.csv("https://ximarketing.github.io/data/banking.csv")
head(mydata)
probit   <- glm(success ~ age + factor(job) + previous, data
                = mydata, family = binomial(link = "probit"))
summary(probit)
```

Using GPT 5:

Run a Probit regression. The dependent variable is success. The independent variables are age, previous, and job. Show the results.



less

复制代码

```less
                      coef    std_err         z     P>|z|    [0.025    0.975]
const              -1.2732     0.0390   -32.958    0.0000   -1.3490   -1.1974
age                 0.0004     0.0009     0.463    0.6430   -0.0012    0.0020
previous            0.4937     0.0139    35.465    0.0000    0.4664    0.5209
job_blue-collar    -0.3122     0.0257   -12.142    0.0000   -0.3627   -0.2617
job_entrepreneur   -0.0364     0.0090    -4.091    0.0000   -0.0539   -0.0189
job_housemaid      -0.0246     0.0101    -2.432    0.0150   -0.0444   -0.0048
job_management     -0.0159     0.0063    -2.528    0.0114   -0.0283   -0.0035
job_retired         0.0694     0.0078     8.925    0.0000    0.0541    0.0847
job_self-employed  -0.0165     0.0085    -1.936    0.0528   -0.0331    0.0001
job_services       -0.0438     0.0059    -7.414    0.0000   -0.0555   -0.0321
job_student         0.0881     0.0088     9.959    0.0000    0.0710    0.1052
job_technician     -0.0164     0.0047    -3.510    0.0004   -0.0256   -0.0072
job_unemployed      0.0081     0.0093     0.868    0.3857   -0.0101    0.0263
job_unknown        -0.0149     0.0170    -0.879    0.3792   -0.0479    0.0181
```

The next question: What should we do when consumers have more than two choices?

More specifically, let us consider the following problem.

Each consumer $i$ has his or her own information, which is measured by the independent variable $X_i$. The dependent variable is a choice made by the consumer, $Y_i \in \{A, B, \ldots\}$.

More specifically, let us consider the following problem.

Each consumer $i$ has his or her own information, which is measured by the independent variable $X_i$. The dependent variable is a choice made by the consumer, $Y_i \in \{A, B, \dots\}$.

Idea: Instead of predicting $Y_i$ directly, we predict the probability $\Pr[Y_i = A], \Pr[Y_i = B], \dots$

Suppose that consumers have three choices, $A, B, C$.

Now, given $X_i$, we would like to come up with three functions $f_A(X_i)$, $f_B(X_i)$ and $f_C(X_i)$, such that

$$\Pr[Y_i = A] \approx f_A(X_i),$$
$$\Pr[Y_i = B] \approx f_B(X_i),$$
$$\Pr[Y_i = C] \approx f_C(X_i).$$

As before, we place a few restrictions on these functions:

1. The probabilities must be nonnegative, i.e., $f_j(X_i) \geq 0$
2. Probabilities cannot exceed 1, i.e., $f_j(X_i) \leq 1$
3. Probabilities are monotone with $X_i$
4. Now, we have a new constraint: all the probabilities must add up to 100%, i.e.,

$$f_A(X_i) + f_B(X_i) + f_C(X_i) = 1.$$

Any ideas for the functions?

```r
library(foreign)
library(nnet)
library(stargazer)
```

We install and load several packages for multinomial logit regression.

# THE MEASUREMENT OF URBAN TRAVEL DEMAND

Daniel McFADDEN*

*Department of Economics, University of California, Berkeley, U.S.A.*

## Part of McFadden's data on route choice

```r
mydata <- read.csv("https://ximarketing.github.io/data/multinomial_route_choice.csv")
head(mydata)
```

## Here is the data...

| | Choice | Flow | Distance | Seat_belt | Passengers | Age | Male | Income | Fuel_efficiency |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Arterial | 460 | 48 | 0 | 0 | 2 | 0 | 1 | 28 |
| 2 | Rural | 440 | 44 | 0 | 0 | 2 | 0 | 1 | 28 |
| 3 | Freeway | 130 | 61 | 0 | 0 | 2 | 0 | 1 | 28 |
| 4 | Arterial | 595 | 59 | 1 | 0 | 2 | 1 | 2 | 27 |
| 5 | Rural | 515 | 70 | 1 | 0 | 2 | 1 | 2 | 27 |
| 6 | Freeway | 340 | 87 | 1 | 0 | 2 | 1 | 2 | 27 |

Here, we want to predict how individuals choose the route when driving. The dependent variable is the chosen route, which can be arterial, rural, and freeway.

The independent variables include the followings:

- Flow: A measure of traffic flow (how busy the traffic is).
- Distance: The distance of the planned trip.
- Seat_belt: whether the driver wears seat belt.
- Passengers: Number of passengers carried.
- Age: Age group of the driver.
- Male: Whether the driver is male or not.
- Income: Income level of the driver.
- Fuel_efficiency: Fuel efficiency level of the vehicle.

We use the multinom function to perform multinomial logit regression:

```
result <- multinom(formula = Choice ~ Flow + Distance +
                    Seat_belt + Passengers + Age + Male +
                    Income + Fuel_efficiency, data = mydata)
result
```

Oh, the results do not read nicely...

```
Coefficients:
        (Intercept)         Flow   Distance  Seat_belt Passengers         Age        Male
Freeway   13.673284 -0.049143703  0.1362782 -0.8924558  0.4775758  0.17728498  0.06331663
Rural      7.558223 -0.008436186 -0.0455514 -0.3451560  0.1436887 -0.06181751 -0.04244764
            Income Fuel_efficiency
Freeway -0.5430466     -0.06321059
Rural    0.1319585     -0.01778424
```

No worries, let's try the stargazer function.

```
stargazer(result, type="html", out="result.html")
```

Now, our results are nicely summarized in the table on the right-hand side:

What does it mean?

| | *Dependent variable:* | |
|---|---|---|
| | Freeway | Rural |
| | (1) | (2) |
| Flow | $-0.049^{***}$ | $-0.008^{***}$ |
| | (0.006) | (0.001) |
| Distance | $0.136^{***}$ | $-0.046^{***}$ |
| | (0.031) | (0.014) |
| Seat_belt | -0.892 | -0.345 |
| | (0.663) | (0.319) |
| Passengers | 0.478 | 0.144 |
| | (0.454) | (0.275) |
| Age | 0.177 | -0.062 |
| | (0.310) | (0.157) |
| Male | 0.063 | -0.042 |
| | (0.638) | (0.302) |
| Income | -0.543 | 0.132 |
| | (0.379) | (0.144) |
| Fuel_efficiency | -0.063 | -0.018 |
| | (0.068) | (0.038) |
| Constant | $13.673^{***}$ | $7.558^{***}$ |
| | (0.158) | (1.390) |
| Akaike Inf. Crit. | 419.424 | 419.424 |
| *Note:* | | $^{*}$p<0.1; $^{**}$p<0.05; $^{***}$p<0.01 |

| | *Dependent variable:* | |
|---|---|---|
| | Freeway | Rural |
| | (1) | (2) |
| Flow | -0.049*** | -0.008*** |
| | (0.006) | (0.001) |
| Distance | 0.136*** | -0.046*** |
| | (0.031) | (0.014) |
| Seat_belt | -0.892 | -0.345 |
| | (0.663) | (0.319) |
| Passengers | 0.478 | 0.144 |
| | (0.454) | (0.275) |
| Age | 0.177 | -0.062 |
| | (0.310) | (0.157) |
| Male | 0.063 | -0.042 |
| | (0.638) | (0.302) |
| Income | -0.543 | 0.132 |
| | (0.379) | (0.144) |
| Fuel_efficiency | -0.063 | -0.018 |
| | (0.068) | (0.038) |
| Constant | 13.673*** | 7.558*** |
| | (0.158) | (1.390) |
| Akaike Inf. Crit. | 419.424 | 419.424 |
| *Note:* | *p<0.1; **p<0.05; ***p<0.01 | |

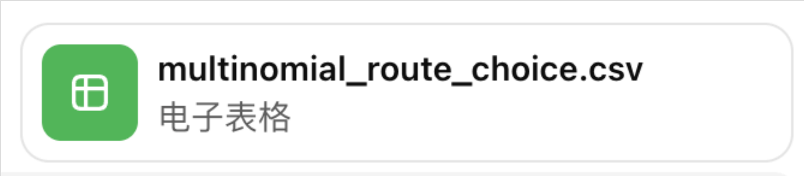
Here, we take arterial as the benchmark and compare other routes against it. Alternatively, you can view the parameters for arterial to be equal to zero.

Flow: When there is a high flow, drivers are very less likely to choose freeway, and a bit less likely to choose rural compared with arterial.

Distance: When distance is long, drivers are more likely to choose freeway and less likely to choose rural route...

Using GPT 5:

I want to run a multinomial logit model. The DV is Choice. All other variables are independent variables. Give me the results.



# The complete code is here:

```r
library(foreign)
library(nnet)
library(stargazer)
mydata <- read.csv("https://ximarketing.github.io/data/multinomial_route_choice.csv")
head(mydata)
result <- multinom(formula = Choice ~ Flow + Distance +
                   Seat_belt + Passengers + Age + Male +
                   Income + Fuel_efficiency, data = mydata)
result
stargazer(result, type="html", out="result.html")
```

# How does AI recognize hand-written digits?

# Conditional Logit Model

In multinomial logit models, a person chooses among a few alternatives. The decision hinges on the decision maker's personal features, not the features of the alternatives. In our previous example, the route decision hinges on features such as distance, age, which are constant across all alternatives.

In conditional logit models, a person chooses among a few alternatives. The decision hinges on the alternatives' features, not the feature of the individuals.

Example:

Consumers choose among three PC brands, A, B, and C.

1. If the choices are based on consumers' age, gender, education etc, then we use the multinomial logit model.
2. If the choices are based on the price, quality of the computers, then we use the conditional logit model.

# Mortgage Data

```r
library(survival)
library(stargazer)
mydata = read.csv("https://ximarketing.github.io/data/conjoint.csv")
head(mydata)
```

| | id | interest | downpayment | rebate | speed | choice |
|---|---|---|---|---|---|---|
| 1 | 1 | 3.75 | 40 | 0.15 | 0.5 | 0 |
| 2 | 1 | 4.00 | 25 | 0.15 | 1.0 | 0 |
| 3 | 1 | 3.75 | 25 | 0.00 | 1.0 | 1 |
| 4 | 2 | 3.50 | 20 | 0.10 | 0.5 | 1 |
| 5 | 2 | 3.75 | 25 | 0.30 | 1.5 | 0 |
| 6 | 2 | 3.75 | 20 | 0.30 | 1.0 | 0 |

| | id | interest | downpayment | rebate | speed | choice |
|---|---|---|---|---|---|---|
| 1 | 1 | 3.75 | 40 | 0.15 | 0.5 | 0 |
| 2 | 1 | 4.00 | 25 | 0.15 | 1.0 | 0 |
| 3 | 1 | 3.75 | 25 | 0.00 | 1.0 | 1 |
| 4 | 2 | 3.50 | 20 | 0.10 | 0.5 | 1 |
| 5 | 2 | 3.75 | 25 | 0.30 | 1.5 | 0 |
| 6 | 2 | 3.75 | 20 | 0.30 | 1.0 | 0 |

Consumer 1 (id = 1) chooses between three mortgage plans:

| Interest Rate | Downpayment | Cash Rebate | Processing Time |
|---|---|---|---|
| 3.75% | 40% | 0.15% | 0.5 month |
| 4.00% | 25% | 0.15% | 1.0 month |
| 3.75% | 25% | 0% | 1.0 month |

This consumer ended up choosing the last plan (choice = 1).

```r
result = clogit(choice ~ interest + downpayment + rebate
                + speed + strata(id), data=mydata)
summary(result)
```

```
                 coef exp(coef)  se(coef)       z Pr(>|z|)    
interest    -1.185055  0.305729  0.097289 -12.181  < 2e-16 ***
downpayment -0.052922  0.948454  0.002336 -22.652  < 2e-16 ***
rebate       0.177522  1.194254  0.149303   1.189  0.23444    
speed       -0.117274  0.889341  0.039587  -2.962  0.00305 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

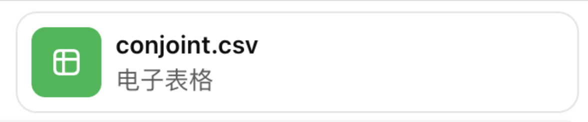
```
stargazer(result, type="html", out="result.html")
```

| | *Dependent variable:* |
|---|---|
| | choice |
| interest | -1.185*** |
| | (0.097) |
| downpayment | -0.053*** |
| | (0.002) |
| rebate | 0.178 |
| | (0.149) |
| speed | -0.117*** |
| | (0.040) |
| Observations | 18,000 |
| $R^2$ | 0.039 |
| Max. Possible $R^2$ | 0.519 |
| Log Likelihood | -6,237.584 |
| Wald Test | 643.940*** (df = 4) |
| LR Test | 708.180*** (df = 4) |
| Score (Logrank) Test | 679.185*** (df = 4) |
| *Note:* | *p<0.1; **p<0.05; ***p<0.01 |

- When interest rate increases, the mortgage is less likely to be chosen;
- When downpayment increases, the mortgage is less likely to be chosen;
- The effect of rebate is not significant;
- When processing time (speed) becomes longer, the mortgage is less likely to be chosen.

Using GPT 5:

I want to run a conditional logit model. id denotes the individual who is making the choice, choice is the person's choice (1 = choose, 0 = not choose), other variables are IVs. Show me the result.



markdown 复制代码

```
                  Conditional Logit Model Regression Results
==============================================================================
Dep. Variable:                  choice   No. Observations:               18000
Model:               ConditionalLogit   No. groups:                      6000
Log-Likelihood:                -6237.6   Min group size:                     3
Method:                           BFGS   Max group size:                     3
Date:                Thu, 04 Dec 2025   Mean group size:                  3.0
Time:                         01:24:50
===============================================================================
                  coef    std err          z      P>|z|      [0.025      0.975]
```

### 📌 Interpretation

- **interest (–1.185, p<0.001):** Higher interest rate makes a product *much less likely to be chosen*.
- **downpayment (–0.053, p<0.001):** Higher required downpayment reduces choice probability.
- **rebate (0.178, ns):** Positive sign, but *not statistically significant*.
- **speed (–0.117, p=0.003):** Higher speed (depending on what it represents in your design) decreases likelihood of being chosen.

# Other Models

There are also many other models beyond ones we discuss in class:

- If your dependent variable is the number of units (e.g., X bottles of milk; Y individuals...), you can use Poisson regression.
- If your dependent variable is censored (e.g., you only observe those whose income is greater than 100K), you can use Tobit model.

https://www.youtube.com/embed/i8tjLQUPc8Y?enablejsapi=1